The general plan

- 1. Our starting point is the set of ordinals below ε_0 .
- 2. Extending this into a continuum of ordinals and defining addition, multiplication and omega powers on them. These will be called *FOATS* ("fractional ordinals at ten").
- 3. Defining continuous fundamental sequences for regular ordinals. Note: *FOATS* do *not* have fundamental sequences.
- 4. Using the above to define a backwards-compatible version of the letters up to *P*, and to define *Q*.
- 5. Outlining a companion array notation to facilitate ease of use.

Fractional Ordinals At Ten (FOATs)

Let $F = \varepsilon_0 \times [0,1)$. An element of *F* be called a "Fractional Ordinal At Ten" (*FOAT*).

Let $\alpha \in \varepsilon_0$, $(\beta, t) \in F$ and $x \in [0, \infty)$. Also, let $I = \lfloor x \rfloor$ and F = x - I. We then define the following operations:

(1) Converting a real number into a *FOAT*:

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RTF(x)=(I,F) (result is a FOAT)
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(2) Addition of an ordinary ordinal and a FOAT:

 $\alpha + (\beta, t) = (\alpha + \beta, t)$ (result is a *FOAT*)

- (3) Multipliation of an omega power and a nonnegative real number:
 - (*i*) $\omega^0 \cdot x = 1 \cdot x = RTF(x)$ (result is a *FOAT*)
 - (*ii*) $\omega^{\alpha} \cdot x = \omega^{\alpha} \cdot I + FS(\omega^{\alpha}, 10F)$ (result is a *FOAT*)

(*FS*(α ,x) basically means "the x-th member of the fundamental sequence of α " and it will be defined in the next section)

(4) Omega powers to *FOATs*:

 $\omega^{(\beta,t)} = \omega^{\beta} \cdot 10^t \qquad (\text{result is a } FOAT)$

(5) Converting a *FOAT* with no fractional part into an ordinal:

 $FTO((\beta,0)) = \beta$ (result is **an ordinal**)

Continuous Fundamental Sequences

Let $E = \varepsilon_0 \cup {\varepsilon_0}$. Then we define a function $FS : E \times [0, \infty) \to F$ as follows:

Let $\alpha \in E$ and $x \in [0,\infty)$. As before, let $I = \lfloor x \rfloor$ and F = x - I. We then define:

- (1) If $\exists \beta, \gamma: \alpha = \beta + \omega^{\gamma} \land \beta \ge \omega^{\gamma}$ then $FS(\alpha, x) = \beta + FS(\omega^{\gamma}, x)$
- (2) $FS(\omega^{\alpha+1},x) = \omega^{\alpha} \cdot x$
- (3) If $\exists \beta < \varepsilon_0 : \alpha = \omega \cdot \beta$ and $x \ge 1$ then $FS(\omega^{\alpha}, x) = \omega^{FS(\alpha, x)}$
- (4) If $\exists \beta < \varepsilon_0 : \alpha = \omega \cdot \beta$ and x < 1 then $FS(\omega^{\alpha}, x) = \omega^{FTO(FS(\alpha, 1))} \cdot x$
- (5) If 0 < x < 1 then $FS(\varepsilon_0, x) = RTF(10^x)$
- (6) If $x \ge 1$ then $FS(\varepsilon_0, x) = \omega^{FS(\varepsilon_0, x-1)}$

Defining Letter Notation Up to Q

Now we are ready to define our notation:

- (1) Valid forms are one of the following: (*i*) $[\alpha]x$ where $\alpha \in E$ and $x \in [0,\infty)$ (*ii*) $[(\beta,t)]10$ where $(\beta,t) \in F$
- (2) $[1]x = 10^x$
- (3) For $x \le 1$: $[\alpha]x = 10^x$
- (4) For x > 1: $[\alpha+1]x = [\alpha][\alpha+1]x-1$
- (5) If $\exists \gamma : \alpha = \omega \cdot \gamma$ and x < 2 then $[\alpha]x = [FTO(FS(\alpha, 2)+1)]x$
- (6) If $\exists \gamma : \alpha = \omega \cdot \gamma$ and $x \ge 2$ then $[\alpha]x = [FS(\alpha, x)]10$
- (7) $[(\beta,t)]10 = [\beta+1](2\cdot 5^t)$
- (8) $Ex = [1]x, Fx = [2]x, Gx = [3]x, Hx = [4]x, Jx = [\omega]x, Kx = [\omega+1]x, Lx = [\omega+2]x,$ $Mx = [\omega\cdot2]x, Nx = [\omega^2]x, Px = [\omega^{\omega}]x, Qx = [\varepsilon_0]x$

Note that if we write:

- (1) $[\omega^{n} \cdot a_{n} + \omega^{n-1} \cdot a_{n-1} + ... + \omega \cdot a_{1} + a_{0}]$ as $[a_{n}, a_{n-1}, ..., a_{1}, a_{0}]$
- (2) $[(\omega^{n} \cdot a_{n} + \omega^{n-1} \cdot a_{n-1} + ... + \omega \cdot a_{1} + a_{0}, t)]$ as $[a_{n}, a_{n-1}, ..., a_{1}, a_{0} + t]$
- (3) $[(\omega^{n} \cdot a_{n} + \omega^{n-1} \cdot a_{n-1} + \dots + \omega^{k} \cdot x)]$ as $[a_{n}, a_{n-1}, \dots, a_{k+1}, x, 0, \dots, 0]$

Then the above definition is backwards-compatible with the previously published array-based definitions.

Associated Array Notation - Extending it to nested arrays

To facilicate calculation and notation, we can write ordinals and *FOATS* as nested arrays:

- (1) The number x is represented by the array 'x'.
- (2) If *A* represents α and *B* represents β then:
 - (*i*) if $\omega^{\omega^{\alpha}} > \beta$ then 'x (A) B' represents $\omega^{\omega^{A}} \cdot x + B$
 - (*ii*) if $\omega^{\omega^{\alpha}} < \beta$ then 'x (A) B' represents $\omega^{\omega^{A}} \cdot B \cdot x$
- (3) If *A* represents α then '1 (*A*)_{*x*} 0' represents $\omega^{\omega^{A,x}}$

This notation doesn't stand on it's own, but it has the advantage of mimicing the behavior of Bird/Bowers nested arrays which many of you are comfortable with.

Example

Let's calculate Q2.021, first by using the concise *FOAT*-based definition and then repeating the calculation more intuitively:

Q2.021 =	$[\varepsilon_0]2.021$	
=	$[FS(\varepsilon_0, 2.021)]10$	(Expansion Rule 6)
=	$[\omega^{FS(\varepsilon_0, 1.021)}]10$	(FS Rule 6)

$ = \begin{bmatrix} \omega^{\mu \sigma T1} (10^{0.61}) 10 & (FS Rule 5) \\ = \begin{bmatrix} \omega^{\mu \sigma T1} (10^{0.6534248952322}) 10 & (Convert Number to FOAT) \\ = \begin{bmatrix} \omega^{\mu 1.10^{0.6564248952322}} 10 & (FOAT power rule) \\ = \begin{bmatrix} \omega^{\mu 1.10^{0.6564248952322}} 10 & (FOAT multiplication rule 2) \\ = \begin{bmatrix} \omega^{\mu 1.1208369216037} 10 & (FOAT multiplication rule 2) \\ = \begin{bmatrix} \omega^{\mu 1.1208369216037} 10 & (FOAT multiplication rule 1) \\ = \begin{bmatrix} \omega^{\mu 1.1208369216037} 10 & (FOAT multiplication rule 1) \\ = \begin{bmatrix} \omega^{\mu 1.100000000000000000000000000000000000$	=	$[\omega^{\omega^{FS(\varepsilon_0,0.021)}}]10$	(FS Rule 6)
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$ \begin{array}{llllllllllllllllllllllllllllllllllll$	=		(FS Rule 2)
= $[\omega^{\omega+1}+(\omega^{\omega}.6+\omega\cdot10^{0.573158948})]10$ (FOAT power rule)= $[\omega^{\omega+1}+(\omega^{\omega}.6+\omega\cdot3.7424753)]10$ (FOAT multiplication rule 2)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega\cdot3+FS(\omega,7.424753))]10$ (FOAT multiplication rule 2)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega\cdot3+1.7.424753))]10$ (FOAT multiplication rule 1)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega\cdot3+7,0.424753))]10$ (FOAT multiplication rule 1)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega\cdot3+7,0.424753))]10$ (Convert Number to FOAT)= $[\omega^{\omega+1}+(\omega^{\omega}.6+\omega\cdot3+7,0.424753)]10$ (FOAT addition rule)= $[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+7,0.424753)]10$ (FOAT addition rule)= $[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+7,0.424753)]10$ (FOAT addition rule)= $[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+7,0.424753)]10$ (Expansion Rule 7)= $[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+8]3.96205$ (Expansion Rule 4)= $[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+7][\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+8]1.96205$ (Expansion Rule 4)= $[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+7]_3[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+8]1.96205$ (Expansion Rule 4)= $[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+7]_3[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+8]1.96205$ (Expansion Rule 4)= $[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+7]_3[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+8]0.96205$ (Expansion Rule 4)	=		(FOAT multiplication rule 1)
= $[\omega^{\omega+1}+(\omega^{\omega}.6+\omega\cdot3.7424753)]10$ (FOAT multiplication rule 2)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega\cdot3+FS(\omega,7.424753))]10$ (FOAT multiplication rule 2)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega\cdot3+1.7.424753))]10$ (FOAT multiplication rule 1)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega\cdot3+7,0.424753))]10$ (Convert Number to FOAT)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega\cdot3+7,0.424753)]10$ (FOAT addition rule)= $[\omega^{\omega+1}+(\omega^{\omega}.6+\omega\cdot3+7,0.424753)]10$ (FOAT addition rule)= $[\omega^{\omega+1}+(\omega^{\omega}.6+\omega\cdot3+7,0.424753)]10$ (FOAT addition rule)= $[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+7].2^{0.424753}$ (Expansion Rule 7)= $[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+8]3.96205$ (Expansion Rule 4)= $[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+7].[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+8]2.96205$ (Expansion Rule 4)= $[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+7].2[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+8]0.96205$ (Expansion Rule 4)= $[\omega^{\omega+1}+\omega^{\omega}.6+\omega\cdot3+7].3.10^{0.96205}$ (Expansion Rule 3)	=	$[\omega^{\omega+1}+(\omega^{\omega}\cdot 6+\omega^{(1,0.573158948)})]10$	(Convert Number to FOAT)
$ = \begin{bmatrix} \omega^{\omega+1} + (\omega^{\omega} \cdot 6 + (\omega \cdot 3 + FS(\omega, 7.424753)))]10 & (FOAT \text{ multiplication rule 2}) \\ = \begin{bmatrix} \omega^{\omega+1} + (\omega^{\omega} \cdot 6 + (\omega \cdot 3 + 1 \cdot 7.424753))]10 & (FS \text{ Rule 2}) \\ = \begin{bmatrix} \omega^{\omega+1} + (\omega^{\omega} \cdot 6 + (\omega \cdot 3 + RTF(7.424753)))]10 & (FOAT \text{ multiplication rule 1}) \\ = \begin{bmatrix} \omega^{\omega+1} + (\omega^{\omega} \cdot 6 + (\omega \cdot 3 + (7, 0.424753)))]10 & (Convert Number to FOAT) \\ = \begin{bmatrix} \omega^{\omega+1} + (\omega^{\omega} \cdot 6 + (\omega \cdot 3 + 7, 0.424753)]10 & (FOAT \text{ addition rule}) \\ = \begin{bmatrix} \omega^{\omega+1} + (\omega^{\omega} \cdot 6 + (\omega \cdot 3 + 7, 0.424753)]10 & (FOAT \text{ addition rule}) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7, 0.424753)]10 & (FOAT \text{ addition rule}) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7, 0.424753)]10 & (FOAT \text{ addition rule}) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]2 \cdot 5^{0.424753} & (Expansion Rule 7) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]3 \cdot 96205 & (Expansion Rule 7) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]2 \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]1 \cdot 96205 & (Expansion Rule 4) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]3 \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]1 \cdot 96205 & (Expansion Rule 4) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]3 \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]1 \cdot 96205 & (Expansion Rule 4) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]3 \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]1 \cdot 96205 & (Expansion Rule 4) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]3 \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]1 \cdot 96205 & (Expansion Rule 4) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]3 \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]1 \cdot 96205 & (Expansion Rule 4) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]3 \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]1 \cdot 96205 & (Expansion Rule 4) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]3 \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]1 \cdot 96205 & (Expansion Rule 4) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]3 \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]1 \cdot 96205 & (Expansion Rule 4) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]3 \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]1 \cdot 96205 & (Expansion Rule 4) \\ = \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]3 \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]1 \cdot 96205 & (Expansion Rule 4) \\ \end{bmatrix} \end{bmatrix} \right)$	=	$[\omega^{\omega+1}+(\omega^{\omega}\cdot 6+\omega\cdot 10^{0.573158948})]10$	(FOAT power rule)
= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega.3+1.7.424753))]10$ (FS Rule 2)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega.3+RTF(7.424753)))]10$ (FOAT multiplication rule 1)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega.3+7,0.424753))]10$ (Convert Number to FOAT)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega.3+7,0.424753)]10$ (FOAT addition rule)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega.3+7,0.424753)]10$ (FOAT addition rule)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega.3+7,0.424753)]10$ (FOAT addition rule)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega.3+7,0.424753)]10$ (Expansion Rule 7)= $[\omega^{\omega+1}+(\omega^{\omega}.6+(\omega.3+8)]2.5^{0.424753})$ (Expansion Rule 7)= $[\omega^{\omega+1}+\omega^{\omega}.6+(\omega.3+8)]3.96205$ (Expansion Rule 4)= $[\omega^{\omega+1}+\omega^{\omega}.6+(\omega.3+7)]_2[\omega^{\omega+1}+\omega^{\omega}.6+(\omega.3+8)]2.96205$ (Expansion Rule 4)= $[\omega^{\omega+1}+\omega^{\omega}.6+(\omega.3+7)]_3[\omega^{\omega+1}+\omega^{\omega}.6+(\omega.3+8)]0.96205$ (Expansion Rule 4)= $[\omega^{\omega+1}+\omega^{\omega}.6+(\omega.3+7)]_3[0^{0.96205}$ (Expansion Rule 4)= $[\omega^{\omega+1}+\omega^{\omega}.6+(\omega.3+7)]_3[0^{0.96205}$ (Expansion Rule 3)	=	$[\omega^{\omega+1}+(\omega^{\omega}\cdot 6+\omega\cdot 3.7424753)]10$	
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$= [\omega^{\omega+1} + (\omega^{\omega} \cdot 6 + (\omega \cdot 3 + (7, 0.424753)))]10 $ (Convert Number to <i>FOAT</i>) $= [\omega^{\omega+1} + (\omega^{\omega} \cdot 6 + (\omega \cdot 3 + 7, 0.424753)]10 $ (<i>FOAT</i> addition rule) $= [\omega^{\omega+1} + (\omega^{\omega} \cdot 6 + \omega \cdot 3 + 7, 0.424753)]10 $ (<i>FOAT</i> addition rule) $= [(\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]2 \cdot 5^{0.424753} $ (Expansion Rule 7) $= [\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]3.96205$ $= [\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7][\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]2.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]_2[\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]1.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]_3[\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]0.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]_3[\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]0.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]_3[\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]0.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]_3[\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]0.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]_3[\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]0.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]_3[\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8]0.96205 $ (Expansion Rule 3)	=	$[\omega^{\omega+1}+(\omega^{\omega}\cdot 6+(\omega\cdot 3+1\cdot 7.424753))]10$	(FS Rule 2)
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$= \begin{bmatrix} \omega^{\omega+1} + (\omega^{\omega} \cdot 6 + \omega \cdot 3 + 7, 0.424753) \end{bmatrix} 10 $ (FOAT addition rule) $= \begin{bmatrix} (\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7, 0.424753) \end{bmatrix} 10 $ (FOAT addition rule) $= \begin{bmatrix} \omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8 \end{bmatrix} 2 \cdot 5^{0.424753} $ (Expansion Rule 7) $= \begin{bmatrix} \omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8 \end{bmatrix} 3.96205 $ (Expansion Rule 4) $= \begin{bmatrix} \omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7 \end{bmatrix} 2 \begin{bmatrix} \omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8 \end{bmatrix} 2.96205 $ (Expansion Rule 4) $= \begin{bmatrix} \omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7 \end{bmatrix} 2 \begin{bmatrix} \omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8 \end{bmatrix} 1.96205 $ (Expansion Rule 4) $= \begin{bmatrix} \omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7 \end{bmatrix} 3 \begin{bmatrix} \omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8 \end{bmatrix} 0.96205 $ (Expansion Rule 4) $= \begin{bmatrix} \omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7 \end{bmatrix} 3 \begin{bmatrix} \omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8 \end{bmatrix} 0.96205 $ (Expansion Rule 4) $= \begin{bmatrix} \omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7 \end{bmatrix} 3 \begin{bmatrix} \omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 8 \end{bmatrix} 0.96205 $ (Expansion Rule 4) $= \begin{bmatrix} \omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7 \end{bmatrix} 3 \begin{bmatrix} 0^{0.96205} \end{bmatrix} $ (Expansion Rule 3)	=	$[\omega^{\omega+1}+(\omega^{\omega}\cdot 6+(\omega\cdot 3+(7,0.424753)))]10$	(Convert Number to FOAT)
$= [(\omega^{\omega+1}+\omega^{\omega}\cdot6+\omega\cdot3+7,0.424753)]10 \qquad (FOAT \text{ addition rule})$ $= [\omega^{\omega+1}+\omega^{\omega}\cdot6+\omega\cdot3+8]2\cdot5^{0.424753} \qquad (Expansion Rule 7)$ $= [\omega^{\omega+1}+\omega^{\omega}\cdot6+\omega\cdot3+8]3.96205$ $= [\omega^{\omega+1}+\omega^{\omega}\cdot6+\omega\cdot3+7][\omega^{\omega+1}+\omega^{\omega}\cdot6+\omega\cdot3+8]2.96205 \qquad (Expansion Rule 4)$ $= [\omega^{\omega+1}+\omega^{\omega}\cdot6+\omega\cdot3+7]_2[\omega^{\omega+1}+\omega^{\omega}\cdot6+\omega\cdot3+8]1.96205 \qquad (Expansion Rule 4)$ $= [\omega^{\omega+1}+\omega^{\omega}\cdot6+\omega\cdot3+7]_3[\omega^{\omega+1}+\omega^{\omega}\cdot6+\omega\cdot3+8]0.96205 \qquad (Expansion Rule 4)$ $= [\omega^{\omega+1}+\omega^{\omega}\cdot6+\omega\cdot3+7]_3[10^{0.96205} \qquad (Expansion Rule 3)$	=	$[\omega^{\omega+1}+(\omega^{\omega}\cdot 6+(\omega\cdot 3+7,0.424753))]10$	(FOAT addition rule)
$= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]2\cdot 5^{0.424753} $ (Expansion Rule 7) $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]3.96205$ $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7][\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]2.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_2[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]1.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_3[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]0.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_310^{0.96205} $ (Expansion Rule 3)	=	$[\omega^{\omega+1}+(\omega^{\omega}\cdot 6+\omega\cdot 3+7,0.424753)]10$	(FOAT addition rule)
$= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]3.96205$ $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7][\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]2.96205 (Expansion Rule 4)$ $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_2[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]1.96205 (Expansion Rule 4)$ $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_3[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]0.96205 (Expansion Rule 4)$ $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_310^{0.96205} (Expansion Rule 3)$	=	$[(\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7,0.424753)]10$	(FOAT addition rule)
$= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7][\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]2.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_2[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]1.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_3[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]0.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_310^{0.96205} $ (Expansion Rule 3)	=	$[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]2\cdot 5^{0.424753}$	(Expansion Rule 7)
$= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_2[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]1.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_3[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]0.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_310^{0.96205} $ (Expansion Rule 3)	=	$[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]3.96205$	
$= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_3[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]0.96205 $ (Expansion Rule 4) $= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_310^{0.96205} $ (Expansion Rule 3)	=	$[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7][\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]2.96205$	(Expansion Rule 4)
$= [\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_3 10^{0.96205} $ (Expansion Rule 3)	=	$[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_2[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]1.96205$	(Expansion Rule 4)
	=	$[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_3[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]0.96205$	(Expansion Rule 4)
$= [\omega^{\omega+1} + \omega^{\omega} \cdot 6 + \omega \cdot 3 + 7]_3 9.163$	=	$[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_310^{0.96205}$	(Expansion Rule 3)
	=	$[\omega^{\omega+1}+\omega^{\omega}\cdot6+\omega\cdot3+7]_39.163$	

=

...

With some practice, you could do the above calculation much more quickly:

Q2.021 =	$[\varepsilon_0]2.021$	
=	$[\omega^{\omega^{10^{0.021}}}]10$	(power tower of <i>int</i> (2.021) ω 's topped by $10^{frac(2.021)}$)
=	$[\omega^{\omega^{1.049542428652322}}]10$	
=	$[\omega^{\omega+1.208369216037}]10$	$(10^{1.0495} = 11.208 = 10^1 + 1.208 \rightarrow \omega^1 + 1.208)$
=	$[\omega^{\omega+1} \cdot 1.61573158948]10$	(10 ^{0.2083} = 1.6157)
=	$[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega^{1.573158948}]10$	$(1.61573 \rightarrow \omega^{\omega+1} \cdot 1 + \omega^{\omega} \cdot 6 + FS(\omega^{\omega}, 1.573))$
=	$[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7.424753]10$	$(10^{1.5731} = 37.424 = 3 \cdot 10^1 + 7.424 \rightarrow \omega^1 \cdot 3 + 7.424)$
=	$[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]2\cdot 5^{0.424753}$	
=	$[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+8]3.96205$	
=	$[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_310^{0.96205}$	(in general $[\alpha+1]x = [\alpha]_{int(x)} 10^{frac(x)}$)
=	$[\omega^{\omega+1}+\omega^{\omega}\cdot 6+\omega\cdot 3+7]_39.163$	

Which can also be written in array notation as $[1,6(1) 3,7]_39.163$