Hypercosmology

Chapter 1

The background theory of Hypercosmology

Members of the V&D and VSBattling community frequently have misconceptions about the structure of hypercosmology. In this chapter the general shape of hypercosmology is outlined and several common misconceptions are cleared up.

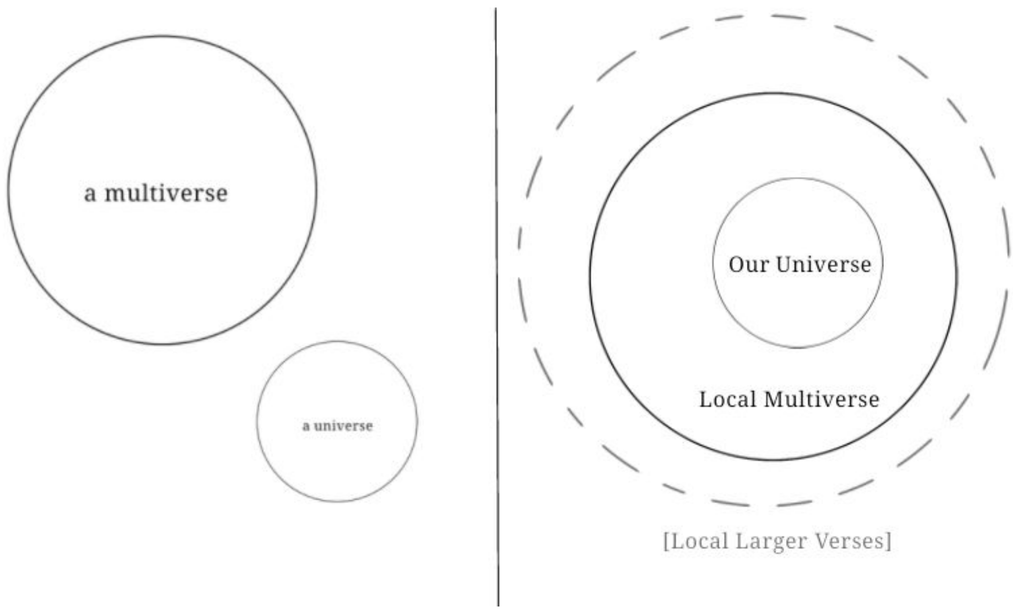
Observable Universe is the name for the universe we currently inhabit. As far as we are aware, it is the only observable space, though this may change in the future. While the entire universe is still a mystery, what we can gather about our observable universe is that it contains 3 spatial dimensions and 1 temporal (time) dimension and is a total of 93 billion light years in diameter. It is possible that the universe is indeed infinite.

However, if our universe is finite it is bound to be smaller than other conceptually possible finite things. Finite numbers which are too big to be expressed by a physical phenomenon within our observable universe are called googological. An example of a very basic googological number is googolplex which is equal to 10^(10^100). Some arguments can be made that googolplex may still have physical meaning within our observable universe, though much greater numbers such as the Graham's number don't have any similar sensible arguments. Googological numbers will be briefly explained in the last section of this chapter.

Recreational Cosmology (collective term for what communities such as AD, V&D and others are engaged in) starts at our universe and goes beyond it

The very first step above the Universe which marks the beginning of (almost) pure speculation is the Multiverse. Different communities define it in different ways which causes inconsistency over the meaning of the term, but V&D's standard definition of the Multiverse is: "a collection of universes, finite or infinite in amount"

A notable misconception is that if a verse is larger than the other then it contains it. That is clearly false because, for example, *a* Multiverse is larger than *a* Universe, but that Universe can be a part of *another* Multiverse and so, not contained by the former Multiverse.
A very useful and related property of verses is locality. If a verse is local to us, it then means that it includes *our* Universe and things "around" it. Only if 2 different verses are both local it follows that the smaller verse is contained in the larger verse.



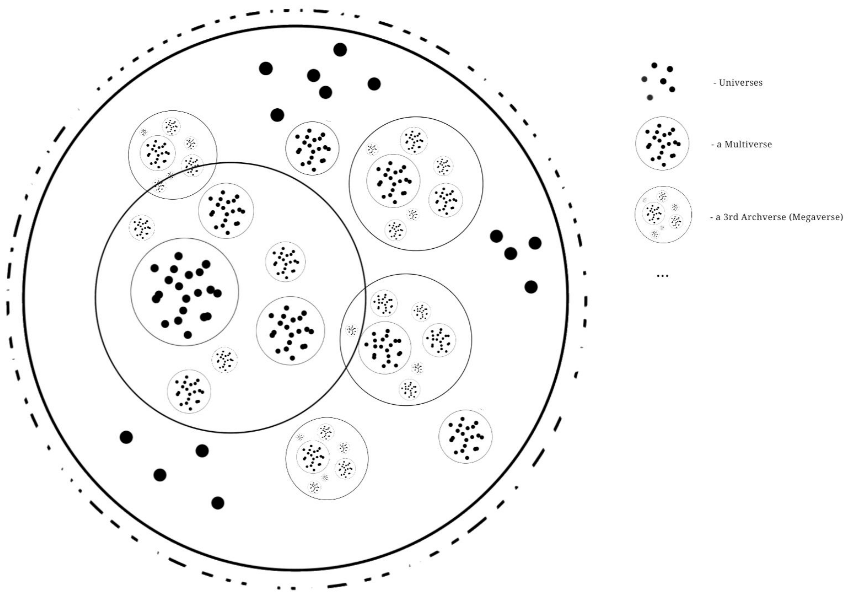
Archverses are the first step at which recursion is applied:

- Universes are taken to be 1st-level Archverses
- Multiverses are taken to be 2nd-level Archverses

and generally,

- n+1st Archverses are defined as a "finite or infinite collection of nth archverses"

[Usually, n+1st Archverses are defined to be larger than nth Archverses by cardinality; what that means will be explained later]



So, for example, Megaverses would be 3rd-level Archverses because they contain finite or infinite amount of Multiverses. The limit of this pattern is achieved at **omniverse** which is the "∞-th Archverse". However it is far from the things which are are considered to be within the realm of Hypercosmology. To go any further a less naive view on infinity is required.

At about the beginning of the 20th century, a mathematician called Georg Cantor defined transfinite numbers which are infinite but can be different from each other in scale/magnitude. There are 2 notions through which the difference can be expressed:

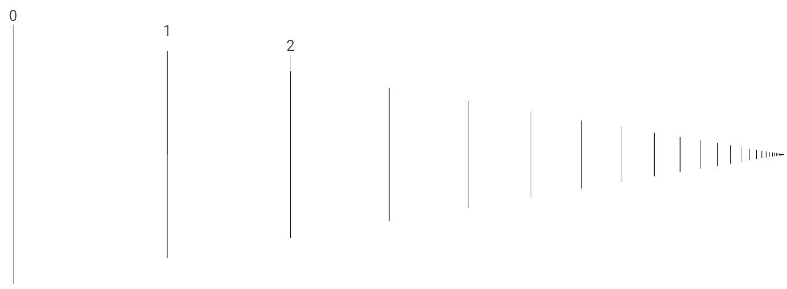
- Cardinality: amount of elements in a set
- Ordinality: position of an element in a set (1st, 2nd..)

Georg Cantor proved that there are infinite sets of different cardinality and that there are orders past "∞th"
To define Archverses bigger than the omniverse, which is the ∞-th Archverse, we need to utilize positions past "∞th"
To do that we need the concept of ordinals. Ordinals are special sets (finite or infinite) which formalize the idea of the position of an element in a set.

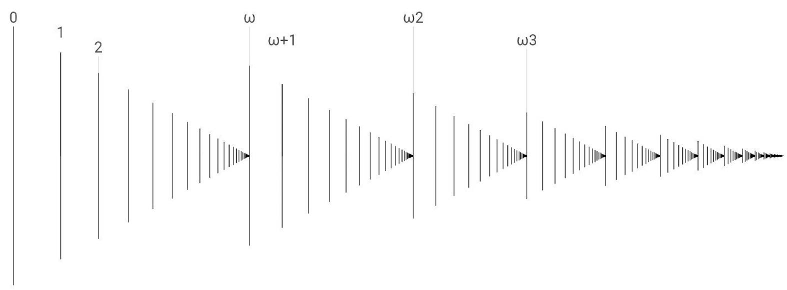
n-th ordinal is the set of all ordinals which are prior to it:

0-th ordinal has no ordinals prior to it so it is the empty set, which is notated as "Ø"
1st ordinal has only 0-th ordinal prior to it, so it is the set which contains only one element - the empty set. 1st ordinal is {Ø}
2nd ordinal has 1st and 0th prior to it so it contains the empty set and the first ordinal. 2nd ordinal is {Ø, {Ø}}
3rd ordinal = {Ø, {Ø}, {Ø, {Ø}}}
4th ordinal = { ∅, {∅} , {∅, {∅}}, {∅, {∅}, {∅, {∅}}} }
...

we arrive at "∞th ordinal" but it is actually well defined, it is simply the set of all finite ordinals, and it is usually notated by the greek letter omega [ω].



ω+1st ordinal is defined as the set which contains all finite ordinals and ω. ω+2nd ordinal is defined as the set which contains all finite ordinals, ω and ω+1. ω+3, ω+4 ... ω+ω, ω*2 + 1, ... ω*2 + 56 ... ω*3, ... ω^ω and so on are defined likewise.



The ordinals defined so far are all <u>countable</u>, meaning that even though the orders are past infinity, the members of an ordinal can be rearranged in such a way that there wouldn't be elements of order past ω-th

for example consider the set
0, 1, 2, 3,...... ω, ω+1, ω+2, ω+3, ... ω+ω

At first glance it seems to be twice the size of regular infinity, but after a simple rearrangement, the set turns into essentially a regular infinite list:

0, ω, 1, $\omega+1$, 2, $\omega+2$, ...

If an infinite set is rearrangeable into such a list it is said that the infinite set is *countable.* Such lists can be enumerated by natural numbers:
1st element corresponds to the natural number "1", 2nd to "2", etc...

However there is an operation that is capable of generating *uncountable* sets - sets which are so large that their members, as a whole, cannot be enumerated by natural numbers (small parts of uncountable sets can still be enumerated but they can never be enumerated as a whole). This operation is called powerset. The powerset operation inputs a set and outputs the set of all of its subsets. The powerset operation is notated as P(x). The powerset of *any* set is *greater* than the original set, this is postulated by Cantor's theorem. It applies even to infinite sets. So, while the set ω is countable, the set $P(\omega)$ is *uncountable* and thus, truly larger than countable sets.

A hierarchy of infinite sets can be formed by consequently applying the powerset operation to ω, and at each step a greater uncountable set is the result.
[note that there may be infinite sets which are between elements of this hierarchy, the assumption that there aren't such sets between ω and $P(\omega)$ is called continuum hypothesis, and the assumption that there aren't such sets between any member of the hierarchy is called generalized continuum hypothesis]

These greater infinite sets, under reasonable axiomatic assumptions, also have ordinal counterparts (ordinals which can be rearranged to resemble them). The ones with the smallest order are called <u>initial ordinals</u> or less formally, and much more frequently, cardinals. The hierarchy of cardinals is the hierarchy of sizes of infinite sets.

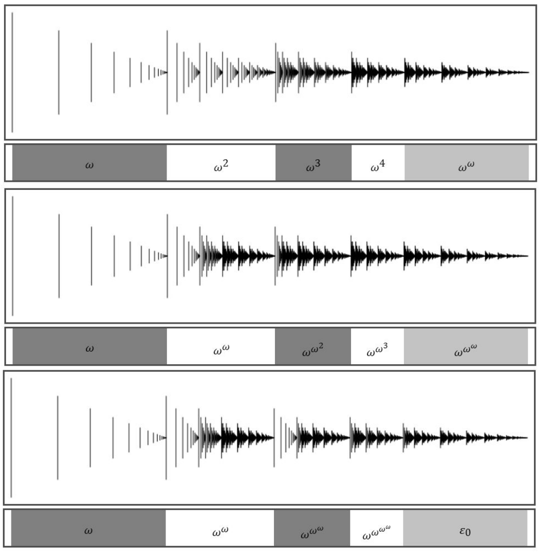
The ordinal ω is the smallest infinite initial ordinal (its cardinal notation is $\aleph_0$)
The ordinal ω_1 is the next infinite initial ordinal (its cardinal notation is $\aleph_1$)
...
ω_2, ω_3 ... ω_ω, $\omega_{\omega+1}$ ... $\omega_{(\omega+\omega)}$ ...... ω_{ω_ω}....
The initial ordinal $\omega_{\omega_{\omega_{\dots}}}$ (its cardinal notation is $\aleph_{\aleph_{\aleph_{\dots}}}$) is called Aleph fixed point.

[ℵ is a letter of Hebrew alphabet called "Aleph"]

Archverses past ω are sometimes called Soupcount verses (named after Soup, who first made a proper hierarchy of archverses using transfinite ordinals).

Note that an ω*2nd Archverse is to omniverse what omniverse is to a Universe. And there are truly huge, yet simply countable ordinals, for example:

ε _0 = ω^ω^ω...
ε _1 = ε _0^ ε _0^ ε _0...
ε _2 = ε _1^ ε _1^ ε _1...
...
ε _ω
....
ε _ ε _0
....
ζ _0 = ε _ ε _ ε _.....



And this is just scratching the very surface of countable ordinals; for each, there is an Archverse.

•1.4. ω. This is the smallest limit ordinal, and the smallest infinite ordinal.

•1.5. $\omega+1$. This is the smallest infinite successor ordinal.

•1.6. $\omega 2$.

•1.7. ω^2.

•1.8. ω^ω.

•1.9. ω^{ω^ω}.

•1.10. $\varepsilon_0 = \varphi(1,0)$. This is the limit of $\omega, \omega^\omega, \omega^{\omega^\omega}, \ldots$, smallest fixed point of $\xi \mapsto \omega^\xi$; in general, $\alpha \mapsto \varepsilon_\alpha = \varphi(1,\alpha)$ is defined as the function enumerating the fixed points of $\xi \mapsto \omega^\xi$. This is the proof-theoretic ordinal of Peano arithmetic.

•1.11. $\varepsilon_1 = \varphi(1,1)$.

•1.12. ε_ω.

•1.13. $\varepsilon_{\varepsilon_0}$.

•1.14. $\varphi(2,0)$. This is the limit of $\varepsilon_0, \varepsilon_{\varepsilon_0}, \ldots$, smallest fixed point of $\xi \mapsto \varepsilon_\xi$; in general, $\alpha \mapsto \varphi(\gamma+1,\alpha)$ is defined as the function enumerating the fixed points of $\xi \mapsto \varphi(\gamma,\xi)$.

•1.15. $\varphi(\omega,0)$. This is the smallest ordinal $> \omega$ closed under primitive recursive ordinal functions ([Avigad2002, corollary 4.5]).

•1.16. The Feferman-Schütte ordinal $\Gamma_0 = \varphi(1,0,0)$ (also $\psi(\Omega^\Omega)$ for an appropriate collapsing function ψ). This is the limit of $\varepsilon_0, \varphi(\varepsilon_0,0), \varphi(\varphi(\varepsilon_0)), \ldots$, smallest fixed point of $\xi \mapsto \varphi(\xi,0)$. This is the proof-theoretic ordinal of ATR_0.

•1.17. The Ackermann ordinal $\varphi(1,0,0,0)$ (also $\psi(\Omega^{\Omega^2})$ for an appropriate collapsing function ψ).

•1.18. The "small" Veblen ordinal ($\psi(\Omega^{\Omega^\omega})$ for an appropriate collapsing function ψ). This is the limit of $\varphi(1,0), \varphi(1,0,0), \varphi(1,0,0,0), \ldots$, the range of the Veblen functions with finitely many variables.

•1.19. The "large" Veblen ordinal ($\psi(\Omega^{\Omega^\Omega})$ for an appropriate collapsing function ψ). This is the range of the Veblen functions with up to that many variables.

•1.20. The Bachmann-Howard ordinal ($\psi(\varepsilon_{\Omega+1})$ for an appropriate collapsing function ψ). This is the proof-theoretic ordinal of Kripke-Platek set theory (KP).

•1.21. The countable collapse of $\varepsilon_{\Omega_\omega+1}$ ("Takeuti-Feferman-Buchholz ordinal"), which is the proof-theoretic ordinal of Π^1_1-comprehension + transfinite induction.

•1.22. The countable collapse of ε_{I+1} where I is the first inaccessible (= Π^1_0-indescribable) cardinal. This is the proof-theoretic ordinal of Kripke-Platek set theory augmented by the recursive inaccessibility of the class of ordinals (KPi), or, on the arithmetical side, of Δ^1_2-comprehension + transfinite induction. See [JaegerPohlers1983]. (Compare •2.3.)

•1.23. The countable collapse of ε_{M+1} where M is the first Mahlo cardinal. This is the proof-theoretic ordinal of KPM. See [Rathjen1990]. (Compare •2.5.)

•1.24. The countable collapse of ε_{K+1} where K is the first weakly compact (= Π^1_1-indescribable) cardinal. This is the proof-theoretic ordinal of $\mathsf{KP}+\Pi_3$-Ref. See [Rathjen1994]. (Compare •2.6.)

•1.25. The countable collapse of $\varepsilon_{\Xi+1}$ where Ξ is the first Π^2_0-indescribable cardinal. This is the proof-theoretic ordinal of $\mathsf{KP} + \Pi_\omega$-Ref. See [Stegert2010, part I] (in whose notation this ordinal would be called $\Psi^{\varepsilon_{\Xi+1}}_{\mathbb{X}}$ where $\mathbb{X} = (\omega^+; \mathsf{P}_0; \epsilon; \epsilon; 0)$).

•••

Countable ordinals within this image are, while truly huge and difficult to comprehend yet still *recursive*, meaning that a finite algorithm can still be created which describes how to reach them starting from ω in finite or transfinite amount of time. For example, to reach ω+1 the algorithm is "add 1"; to reach ω+ω the algorithm is "keep adding 1"; to reach ε _0 the algorithm is "start at ω and every time you apply the "^ω" operation add 1 to a counter, when the counter reaches ω - stop". The larger a countable ordinal is, the more complex algorithms are. However, there is a smallest countable ordinal unreachable by any algorithm no matter how complex it is. The ordinal is called Church-Kleene ordinal and is notated as ω1ck.

It is incomparably larger than any *recursive* ordinals.
If we allow algorithms to start at ω1ck instead of ω. we will be limited by the second ordinal which is unreachable from below. We can apply recursion to the definition of unreachability and then we will still be unable to reach some other greater countable ordinal. Generalizing this we can form even greater hierarchies of unreachability. Set theorists have defined many such extensions [see the image below]

•2.1. The Church-Kleene ordinal ω_1^{CK}: the smallest admissible ordinal $> \omega$. This is the smallest ordinal which is not the order type of a recursive (equivalently: hyperarithmetic) well-ordering on ω. The ω_1^{CK}-recursive (resp. ω_1^{CK}-semi-recursive) subsets of ω are exactly the Δ_1^1 (=hyperarithmetic) (resp. Π_1^1) subsets of ω, and they are also exactly the subsets recursive (resp. semi-recursive) in E (or $E^\#$, CHECK THIS[this is stated vaguely and without proof in [HinmanMoschovakis1971, §2, introductory remarks], and also alluded to, but with an argument, in [Hinman1978, chapter VI, introductory remarks to §6 on p. 316]; but the essential argument should be Gandy's selection theorem, [Hinman1978, chapter VI, theorem 4.1 on p. 292 or its corollary 4.3 on p. 294]]).

•2.2. ω_ω^{CK}: the smallest limit of admissibles. This ordinal is not admissible. This is the smallest α such that $L_\alpha \cap \mathscr{P}(\omega)$ is a model of Π_1^1-comprehension (cf. [Simpson2009, theorem VII.1.8 on p. 246 and theorem VII.5.17 on p. 292 and notes to §VII.5 on p. 293]).

•2.3. The smallest recursively inaccessible ordinal: this is the smallest ordinal which is admissible and limit of admissibles. This is the smallest ordinal α such that $L_\alpha \models \mathsf{KPi}$, or, on the arithmetical side, such that $L_\alpha \cap \mathscr{P}(\omega)$ is a model of Δ_2^1-comprehension (cf. [Simpson2009, theorem VII.3.24 on p. 267 and theorem VII.5.17 on p. 292 and errata[1] to notes to §VII.5 on p. 293]). (Compare •1.22.)

This is the smallest ordinal $\omega_1^{E_1}$ not the order type of a well-ordering recursive in the Tugué functional E_1 ([Hinman1978, chapter VIII, theorem 6.6 on p. 421]), or equivalently, recursive in the hyperjump; and for this α the α-recursive (resp. α-semi-recursive) subsets of ω are exactly the subsets recursive (resp. semi-recursive) in E_1 ([Hinman1978, chapter VIII, corollary 4.16 on p. 412]).

This is the smallest α such that $L_\alpha \models \mathsf{KP} + \mathit{Beta}$, where Beta asserts the existence of a transitive collapse for any well-founded relation, or equivalently, the smallest admissible α such that any ordering which L_α thinks is a well-ordering is, indeed, a well-ordering: see [Nadel1973, theorem 6.1 on p. 291] (compare [Harrison1968] for the negative result concerning the ordinal ω_1^{CK} of •2.1; compare also [Gostanian1979] and •2.9 for related facts).

•2.4. The smallest recursively hyperinaccessible ordinal: i.e., the smallest recursively inaccessible which is a limit of recursively inaccessibles.

•2.5. The smallest recursively Mahlo ordinal: i.e., the smallest admissible ordinal α such that for any α-recursive function $f\colon \alpha \to \alpha$ there is an admissible $\beta < \alpha$ which is closed under f. This is the smallest ordinal α such that $L_\alpha \models \mathsf{KPM}$. (Compare •1.23.)

This is the smallest ordinal not the order type of a well-ordering recursive in the superjump ([AczelHinman1974] and [Harrington1974]); and for this α the α-recursive (resp. α-semi-recursive) subsets of ω are exactly the subsets recursive in the superjump (resp. semirecursive in the partial normalization of the superjump, [Harrington1974, theorem 5 on p. 50]).

Also note concerning this ordinal: [RichterAczel1974, corollary 9.4(ii) on p. 348].

•2.6. The smallest Π_3-reflecting ("recursively weakly compact") ordinal. This can also be described as the smallest "2-admissible" ordinal: see [RichterAczel1974, theorem 1.16 on p. 312]. (Compare •1.24.)

Also the sup of the closure ordinals for Σ_3 inductive operators: [RichterAczel1974, theorem A on p. 303]. For this α the α-semi-recursive subsets of ω are exactly the Σ_3-inductively definable subsets of ω ([RichterAczel1974, theorem A on p. 303 and theorem D on p. 304]; see also [Simpson1978, example 4.12 on p. 370]).

•2.7. The smallest $(+1)$-stable ordinal, i.e., the smallest α such that $L_\alpha \preceq_1 L_{\alpha+1}$. This is the smallest Π_0^1-reflecting (i.e., Π_n-reflecting for every $n \in \omega$) ordinal: [RichterAczel1974, theorem 1.18 on p. 313 and 333].

(Compare •1.25.)

•2.8. The smallest $(^+)$-stable ordinal, i.e., the smallest α such that $L_\alpha \preceq_1 L_{\alpha^+}$ where α^+ is the smallest admissible ordinal $> \alpha$. This is the smallest Π_1^1-reflecting ordinal: [RichterAczel1974, theorem 1.19 on p. 313 and 336]. Also the sup of the closure ordinals for Π_1^1 inductive operators: [RichterAczel1974, theorem B on p. 303 or 10.7 on p. 355] and [Cenzer1974, theorem A on p. 222]. For this α the α-semi-recursive subsets of ω are exactly the Π_1^1-inductively definable subsets of ω ([RichterAczel1974, theorem D on p. 304]; see also [Simpson1978, example 4.13 on p. 370]).

This is the smallest ordinal $\omega_1^{G_1^\#}$ not the order type of a well-ordering recursive in the nondeterministic functional $G_1^\#$ defined by $G_1^\#(f) \approx \{f(0)\}_{(\omega_1^f)^+}(f(1))$; and for this α the α-recursive (resp. α-semi-recursive) subsets of ω are exactly the subsets recursive (resp. semi-recursive) in $G_1^\#$ ([Cenzer1974, theorem 7.4 on p. 238]).

•2.9. The smallest Σ_1^1-reflecting ordinal. Also the sup of the closure ordinals for Σ_1^1 inductive operators: [RichterAczel1974, theorem B on p. 303 or 10.7 on p. 355]. For this α the α-semi-recursive subsets of ω are exactly the Σ_1^1-inductively definable subsets of ω ([RichterAczel1974, theorem D on p. 304]; see also [Simpson1978, example 4.14 on p. 370]).

That this ordinal is greater than that of •2.8: [Aanderaa1974, corollary 1 to theorem 6 on p.213]; also see: [Simpson1978, theorem 6.5] and [GostanianHrbáček1979].

This is the smallest ordinal $\omega_1^{E_1^\#}$ not the order type of a well-ordering recursive in the nondeterministic version $E_1^\#$ of the Tugué functional E_1; and for this α the α-recursive (resp. α-semi-recursive) subsets of ω are exactly the subsets recursive (resp. semi-recursive) in $E_1^\#$ (combine [Aczel1970, theorem 1 on p. 313, theorem 2 on p. 318] and [RichterAczel1974, theorem D on p. 304]).

This is the smallest admissible α which is not Gandy, i.e., such that every α-recursive linear ordering of α of which L_{α^+} thinks that it is a well-ordering (with α^+ being the next admissible) is, indeed, a well-ordering: see [Simpson1978, theorem 6.6 on p. 377] and [Gostanian1979, theorem 3.3] (on the terminology "Gandy ordinal", see [AbramsonSacks1976]: in [Gostanian1979] the same ordinals are called "good").

•2.10. The smallest $(^{++})$-stable ordinal, i.e., the smallest α such that $L_\alpha \preceq_1 L_{\alpha^{++}}$ where α^+, α^{++} are the two smallest admissible ordinals $> \alpha$. This is Σ_1^1-reflecting and greater than the ordinal of •2.9 ([Simpson1978. theorem 6.4 on p. 376] and proposition 3.1 below).

•2.11. The smallest inaccessibly-stable ordinal, i.e., the smallest α such that $L_\alpha \preceq_1 L_\beta$ where β is the smallest recursively inaccessible (cf. •2.3) ordinal $> \alpha$.

•2.12. The smallest Mahlo-stable ordinal, i.e., the smallest α such that $L_\alpha \preceq_1 L_\beta$ where β is the smallest recursively Mahlo (cf. •2.5) ordinal $> \alpha$.

•2.13. The smallest doubly $(+1)$-stable ordinal, i.e., the smallest α such that $L_\alpha \preceq_1 L_\beta \preceq_1 L_{\beta+1}$ (cf. •2.7).

•2.14. The smallest stable ordinal under a nonprojectible ordinal, i.e., the smallest α such that $L_\alpha \preceq_1 L_\beta$ where β is the smallest nonprojectible (the ordinal of •2.15).

This is the smallest ordinal $\omega_1^{\mathbf{R}}$ not the order type of a well-ordering recursive in a certain type 3 functional $\mathbf{R}$ defined in [Harrington1975]; and for this α the α-recursive subsets of ω are exactly the subsets recursive in $\mathbf{R}$. (See also [John1986] and [Simpson1978, example 4.10 on p. 369].)

•2.15. The smallest nonprojectible ordinal, i.e., the smallest β such that β is a limit of β-stable ordinals (ordinals α such that $L_\alpha \preceq_1 L_\beta$ (cf. •2.14); in other words, the smallest β such that $L_\beta \models \mathsf{KPi}$+"the stable ordinals are unbounded". This is the smallest ordinal β such that $L_\beta \models \mathsf{KP}\omega + \Sigma_1\text{-}\mathsf{Sep}$ (cf. [Barwise1975, chapter V, theorem 6.3 on p. 175]), or such that $L_\beta \cap \mathscr{P}(\omega)$ is a model of Π_2^1-comprehension (cf. [Simpson2009, theorem VII.3.24 on p. 267 and theorem VII.5.17 on p. 292]).

In Jensen's terminology ([Jensen1972]), this is the smallest ordinal β such that $\rho_1^\beta > \omega$, and in fact the smallest $\beta > \omega$ such that $\rho_1^\beta = \beta$: that is, the smallest ordinal β such that every $\Sigma_1(L_\beta)$ subset of ω is β-finite. Sometimes also called the smallest "strongly admissible" (or "strongly Σ_1-admissible") ordinal.

•2.16. The smallest (weakly) Σ_2-admissible ordinal. This is the smallest ordinal β such that $L_\beta \models \mathsf{KP}\omega + \Delta_2\text{-}\mathsf{Sep}$, or such that $L_\beta \cap \mathscr{P}(\omega)$ is a model of Δ_3^1-comprehension (cf. [Simpson2009, theorem VII.3.24 on p. 267 and theorem VII.5.17 on p. 292]).

In Jensen's terminology ([Jensen1972]), this is the smallest ordinal β such that $\eta_2^\beta > \omega$, and in fact the smallest $\beta > \omega$ such that $\eta_2^\beta = \beta$: that is, the smallest ordinal β such that every $\Delta_2(L_\beta)$ subset of ω is β-finite.

•2.17. The ordinal of ramified analysis (often written β_0). This is the smallest β such that $L_\beta \models \bigwedge_n \Sigma_n\text{-}\mathsf{Sep}$ (the full separation scheme), or such that $L_\beta \cap \mathscr{P}(\omega)$ is a model of full second-order analysis (second-order comprehension), and in fact $L_\beta \models \mathsf{ZFC}^-$ (that is, ZFC minus the powerset axiom).

This starts the first gap in the constructible universe, and this gap is of length 1: see [Putnam1963] and [MarekSrebrny1973, corollary 4.5 on p. 374].

Note that this ordinal is $(+1)$-stable (cf. •2.7) but not $(+2)$-stable: [MarekSrebrny1973, corollary to theorem 6.14 on p. 384].

•2.18. The start of the first gap of length 2 in the constructible universe. If β is this ordinal then β is the β-th gap ordinal ([MarekSrebrny1973, theorem 4.17 on p. 377]).

•2.19. The first ordinal β which starts a gap of length β in the constructible universe.

•2.20. The ordinal $\beta = \omega_1^{L_\alpha}$ where α is ordinal of •2.21. Then by construction β starts a gap of length $\alpha = \beta^+$ (the next admissible ordinal).

•2.21. The smallest ordinal α such that $L_\alpha \models \mathsf{KP}$+"$\omega_1$ exists", i.e., the smallest admissible α which is not locally countable, or equivalently, the smallest α such that $L_\alpha \models \mathsf{KP}$+"$\mathscr{P}(\omega)$ exists" (cf. proposition 3.2).

•2.22. The smallest ordinal α such that $L_\alpha \models \mathsf{ZFC}^-$+"$\omega_1$ exists", or equivalently such that $L_\alpha \models \mathsf{ZFC}^-$+"$\mathscr{P}(\omega)$ exists" (cf. proposition 3.2). This is the start of the first third-order gap ([MarekSrebrny1973, theorem 3.7 on p. 372]) in the constructible universe.

•2.23. The smallest uncountable ordinal $\omega_1^{L_\alpha}$ in the smallest model L_α of ZFC, assuming it exists (see •2.24). This ordinal is α-stable.

•2.24. The smallest ordinal α such that $L_\alpha \models \mathsf{ZFC}$ (assuming it exists), i.e., the height of the minimal model of ZFC.

•2.25. The smallest stable ordinal σ, i.e., the smallest σ such that $L_\sigma \preceq_1 L$, or equivalently $L_\sigma \preceq_1 L_{\omega_1}$. The set L_σ is the set of all x which are Σ_1-definable in L without parameters ([Barwise1975, chapter V, corollary 7.9(i) on p. 182]).

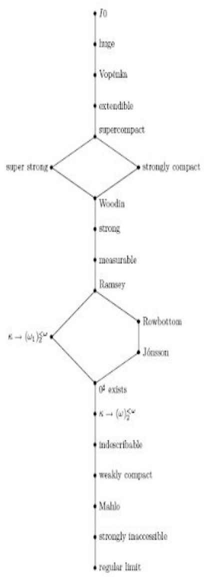
This ordinal is projectible to ω (i.e., in Jensen's terminology), $\rho_1^\sigma = \omega$ ([Barwise1975, chapter V, theorem 7.10(i) on p. 183]).

This is the smallest ordinal δ_2^1 which not the order type of a well-ordering Δ_2^1 on ω; and in fact, for this $\sigma = \delta_2^1$, the σ-recursive (resp. σ-semi-recursive) subsets of ω are exactly the Δ_2^1 (resp. Σ_2^1) subsets of ω ([Barwise1975, chapter V, theorem 8.2 on p. 189 and corollary 8.3 on p. 191]).

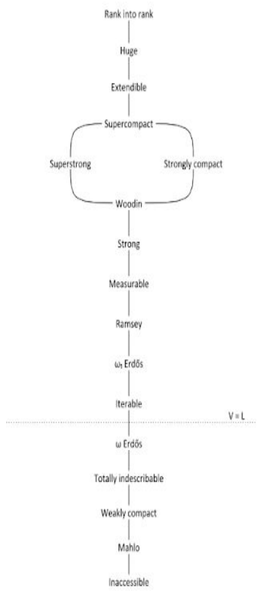
This is also the smallest Σ_2^1-reflecting ordinal ([Richter1975]).

There are special cardinals that are so big that regular set-theory axioms (which are more than enough to establish regular mathematics) can't establish their existence. These are called large cardinals. As a consequence of their sheer size and yet pure formality their definitions are very technical and sophisticated. Their scale is incomparably greater than the scale of previously defined ordinals:

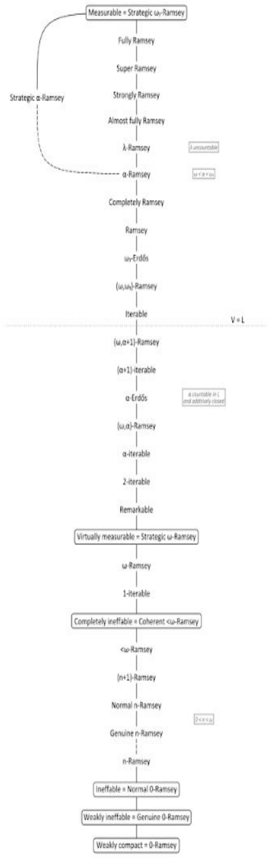
Strongly Inaccessible cardinal: a cardinal which cannot be reached by powerset hierarchies and permutations and strengthenings of the hierarchies which are definable in ZFC set theory
…
2nd strongly inaccessible cardinal is in the same sense unreachable even if we have 1st strongly inaccessible cardinal as a starting point.
...
3rd inaccessible, … ωth .... inaccessible-th, 2-inaccessible, hyper-inaccessible, hyper-hyper-inaccessible…]
…
Mahlo cardinal
………..
Weakly compact cardinal
……………[many other much greater cardinals set theorists have considered, see next page]



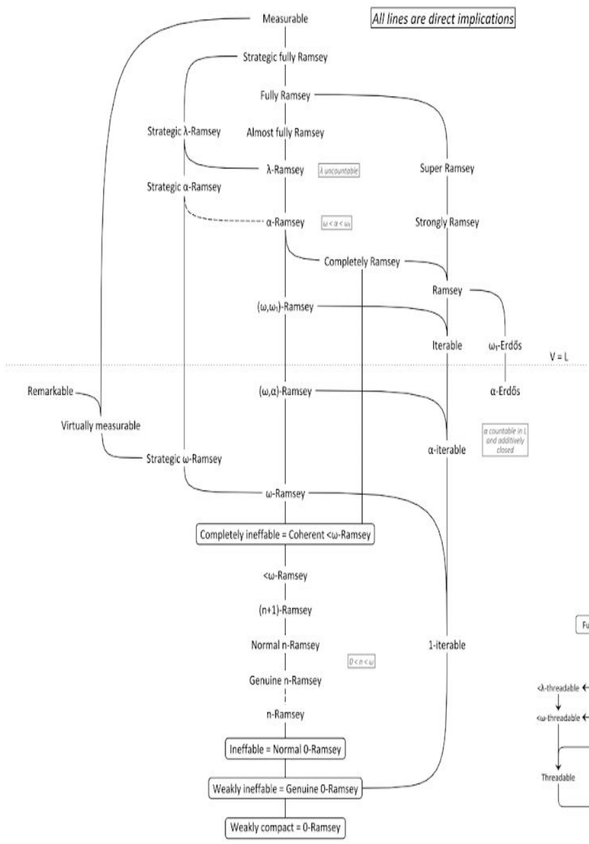
I0
I2
Vopěnka
extendible
supercompact
super strong
strongly compact
Woodin
strong
measurable
Ramsey
Rowbottom
$\kappa \to (\omega_1)_2^{<\omega}$
Jónsson
$0^\sharp$ exists
$\kappa \to (\omega)_2^{<\omega}$
indescribable
weakly compact
Mahlo
strongly inaccessible
regular limit



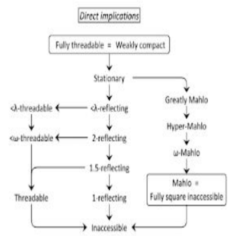
Rank into rank
Huge
Extendible
Supercompact
Superstrong
Strongly compact
Woodin
Strong
Measurable
Ramsey
ω_1 Erdős
Iterable
V = L
ω Erdős
Totally indescribable
Weakly compact
Mahlo
Inaccessible



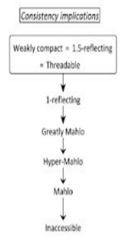
Measurable = Strategic ω_1-Ramsey
Fully Ramsey
Super Ramsey
Strategic α-Ramsey
Strongly Ramsey
Almost fully Ramsey
λ-Ramsey
λ uncountable
α-Ramsey
Completely Ramsey
Ramsey
ω_1-Erdős
(ω,ω_1)-Ramsey
Iterable
V = L
(ω,α+1)-Ramsey
(α+1)-iterable
α-Erdős
α countable in L and additively closed
(ω,α)-Ramsey
α-iterable
2-iterable
Remarkable
Virtually measurable = Strategic ω-Ramsey
ω-Ramsey
1-iterable
Completely ineffable = Coherent <ω-Ramsey
<ω-Ramsey
(n+1)-Ramsey
Normal n-Ramsey
Genuine n-Ramsey
n-Ramsey
Ineffable = Normal 0-Ramsey
Weakly ineffable = Genuine 0-Ramsey
Weakly compact = 0-Ramsey



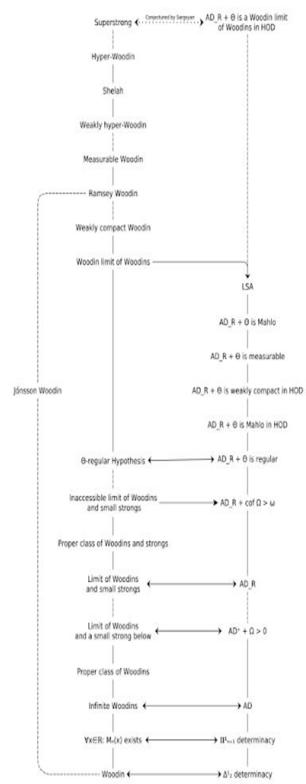
Measurable
All lines are direct implications
Strategic fully Ramsey
Fully Ramsey
Strategic λ-Ramsey
Almost fully Ramsey
λ-Ramsey
λ uncountable
Super Ramsey
Strategic α-Ramsey
α-Ramsey
$\omega < \alpha < \omega_1$
Strongly Ramsey
Completely Ramsey
Ramsey
(ω,ω_1)-Ramsey
Iterable
ω_1-Erdős
V = L
Remarkable
(ω,α)-Ramsey
α-Erdős
Virtually measurable
α countable in L and additively closed
α-iterable
Strategic ω-Ramsey
ω-Ramsey
Completely ineffable = Coherent <ω-Ramsey
<ω-Ramsey
(n+1)-Ramsey
Normal n-Ramsey
$0 < n < \omega$
1-iterable
Genuine n-Ramsey
n-Ramsey
Ineffable = Normal 0-Ramsey
Weakly ineffable = Genuine 0-Ramsey
Weakly compact = 0-Ramsey



Direct implications
Fully threadable = Weakly compact
Stationary
Greatly Mahlo
<λ-threadable
<λ-reflecting
Hyper-Mahlo
<ω-threadable
2-reflecting
ω-Mahlo
1.5-reflecting
Mahlo = Fully square inaccessible
Threadable
1-reflecting
Inaccessible



Consistency implications
Weakly compact = 1.5-reflecting = Threadable
1-reflecting
Greatly Mahlo
Hyper-Mahlo
Mahlo
Inaccessible



Superstrong
Conjectured by Sargsyan
AD_R + Θ is a Woodin limit of Woodins in HOD
Hyper-Woodin
Shelah
Weakly hyper-Woodin
Measurable Woodin
Ramsey Woodin
Weakly compact Woodin
Woodin limit of Woodins
LSA
AD_R + Θ is Mahlo
AD_R + Θ is measurable
Jónsson Woodin
AD_R + Θ is weakly compact in HOD
AD_R + Θ is Mahlo in HOD
Θ-regular Hypothesis
AD_R + Θ is regular
Inaccessible limit of Woodins and small strongs
AD_R + cof Ω > ω
Proper class of Woodins and strongs
Limit of Woodins and small strongs
AD_R
Limit of Woodins and a small strong below
AD⁺ + Ω > 0
Proper class of Woodins
Infinite Woodins
AD
∀x∈R: $M_n(x)$ exists
Π^1_{n+1} determinacy
Woodin
Δ^1_2 determinacy

For each of the cardinals [and many, many ordinals between them] Archverses can be defined. These massive Archverses completely dwarf anything previously defined. It is truly difficult to pinpoint the extreme differences in scale.

We are now entering the realm of low-tier Hypercosmology. At this level even highly abstract mathematical modeling of the concepts involved is highly problematic.

Ordverse is defined after Ord - the class of all ordinals. It is the limit of all Archverses indexed by transfinite ordinals.

Past that we can use proper higher order collections to extend the notion of ordinals past the concept of sets. Proper 1st order collections are sets. Proper 2nd order collections are proper classes which are too big to be sets (they would be as big as the set of all sets, if such sets weren't contradictory). proper 3rd order collections are proper conglomerates which are too big to be proper classes, and so on…
At each level there exist more and more ordinal-like structures called proper collection-ordinals.

The hierarchy can be iterated into transfinite, and then further.
Within the hierarchy ridiculously high orders such as "Properclass-order proper collections" and "Properclass-order proper collection-order proper collections" are defined. And for each of those ridiculous levels proper collection-ordinals are also defined. At this point their formalizability and consistency is becoming a significant issue.

Just like with regular ordinals, proper collection-ordinals can be used to define Archverses. The limit of such Archverses which are local is the Barrelplex. It is V&D's local Altarca.

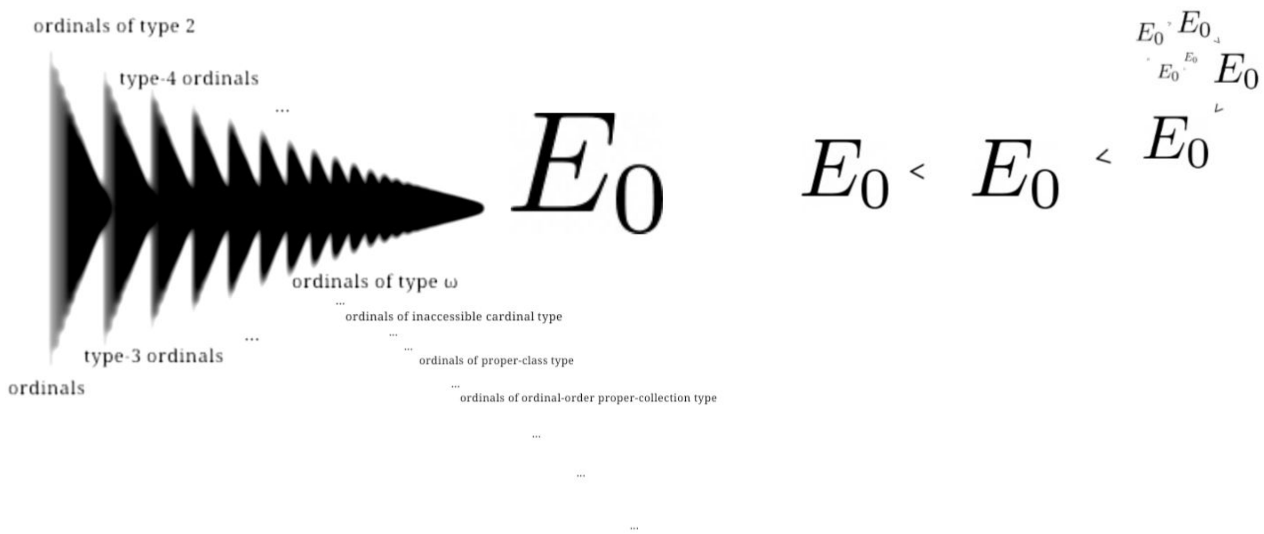
Altarca is a cosmology which doesn't exceed Barrelplex in size and as a consequence of that, can be ordered using ordinals or proper collection-ordinals.

After all hopes of strict, concrete and consistent mathematical formalization are lost we get into lower mid-tier hypercosmology. This tier uses to its advantage broad, informal reasoning to establish results. [Purely theoretical possibility of formalization still isn't fully rejected at this level, however]

A very common misconception is that contradictions are in the realm of utter illogic and chaos. It is however very untrue. Only within the domain of classical logic are contradictions an "all-shattering" catastrophe. Systems of logic which are paraconsistent reject the law that from contradiction everything follows, and as a consequence tolerate contradictions, usually at the cost of deductive strength. There are other forms of non-classical logic which may be even more otherworldly than paraconsistent logic, yet they are all valid forms of reasoning in their respective domains.

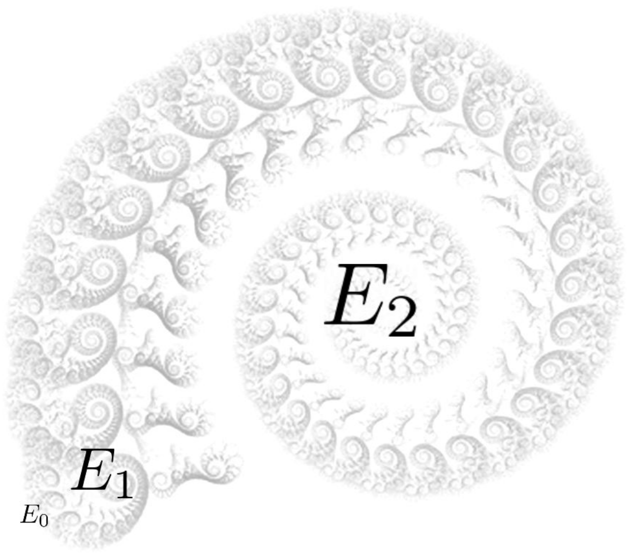
Post-infinite numbers are a type of numbers that is too pathological to be precisely and formally defined. These numbers are greater than all collections, and so are greater than all orders. As a consequence they are too big to be properly ordered.

E_0 is the smallest post-infinite number, it comes after all numbers which aren't greater than themselves. It is the "size" of the pseudo-collection of *all* collections. [E_0-verse is hardly even an archverse and is more frequently called the 1st Selfverse, it is called so because it is greater than itself]



E_1 is defined as the first number that is larger than E_0 and isn't simultaneously equal to it. The *degree at which it is greater than itself* is greater than that of E_0

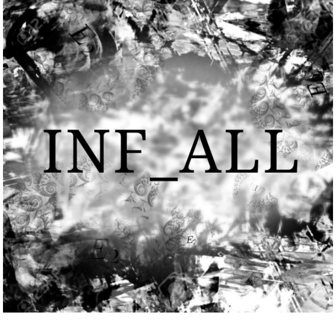
E numbers can be indexed by ordinals, proper collection-ordinals or even other E numbers. For each E number an E-th "Archverse" can be defined.
"Archverses" which have levels greater than E_0 are called Selfverses. These verses contain more than they themselves contain.



[This image is purely an artistic, metaphorical expression of the post-infinite E numbers. the structure on the image itself is a simple fractal, which is easy to formalize within regular mathematical systems]

The greater a post-infinite number is, the more order-related pathologies occur. At first, linearity of the hierarchy is lost as post-infinite numbers begin to form groups where they are each greater than each other within the group. Then even weaker forms of orderings are lost.

As a consequence of that, arithmetic on such numbers (if at all definable) is currently completely unknown.



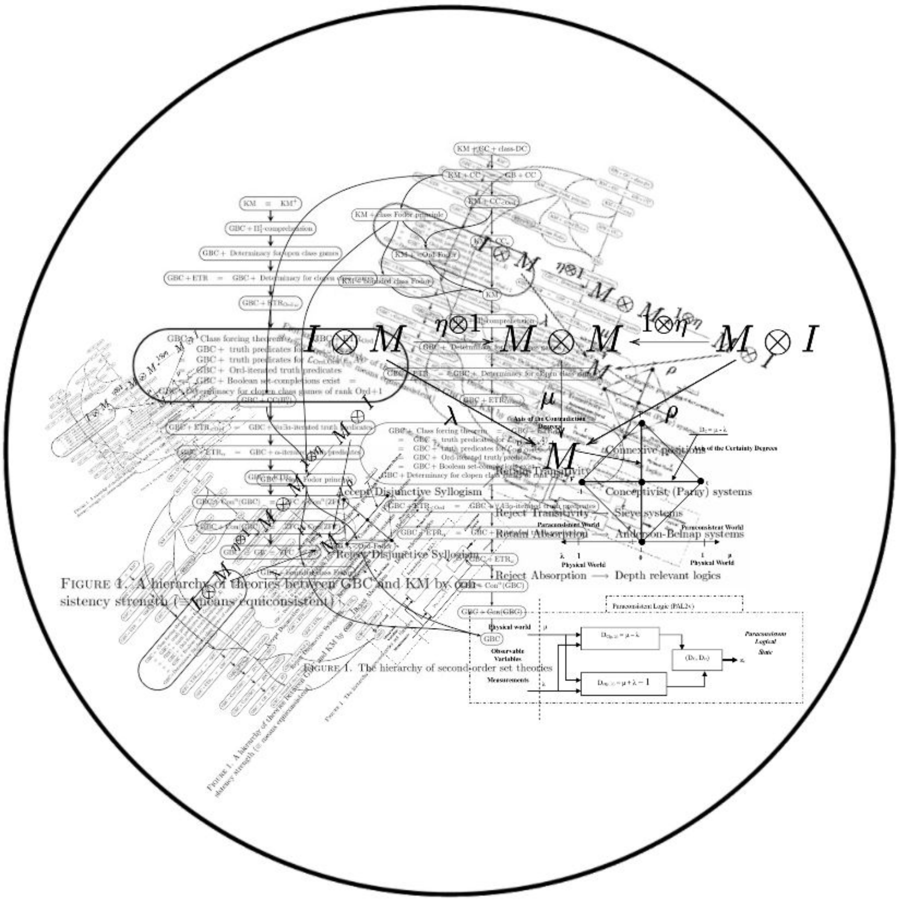
The limit of all post-infinites is a number called INF_ALL. It is the number at which any remnants of a structured hierarchy are lost. Its verse-counterpart is called Maiorverse which is also the limit of all post-infinite indexed "Archverses".
[In theory, extensions such as "Archverses past Maiorverse" are possible but such extensions require considerably more effort and are very technical and quite fruitless.]

Classical Logic Verse: the verse which contains everything which satisfies:

- The law of Excluded Middle: everything is either true or false
- The law of Noncontradiction: nothing contradicts itself
- The principle of Identity: everything is equal to itself

Intermediate Logic-Verses are associated with logics which are less restrictive such as logics which Intuitionistic or Paraconsistent.

Formalverse: the verse which contains *everything* formal or, in theory, *eventually* formalizable. It is conjectured to be above Maiorverse.



Artistic impression of Formalverse

Now we are entering mid-tier Hypercosmology. Formalization at this point (due to being above Formalverse), even in theory, is completely impossible. Very careful and systematic yet purely informal reasoning is required to establish results at this level.

A very important principle of hypercosmology is that the more informal a logic is, the more powerful and expressive it is, at the cost of being vague and imprecise.

Binaryfield: contains everything that is or isn't a random object/concept (such as a potato)

Binaryfield is clearly larger than Maiorverse or Formalverse because it also contains everything that isn't them. However, due to nonclassical logic, Binaryfield isn't absolutely everything, because such logics allow things to be neither themselves nor not themselves. In fact, the very limitation of things to either be or not to be, is, from the point of view of nonclassical logics, arbitrary and excessively strict, so there are *many more* things outside Binaryfield, though very strange things, from the human point of view.

[Note that Formalverse contains things that are neither themselves nor not themselves from the *formal* point of view due to *formal* nonclassical logics; informally, those are still themselves, and so are within Binaryfield]

Trinaryfield: contains everything within Binaryfield and outside Binaryfield. It again may seem that this is absolutely everything, but again, the restriction to either be within Binaryfield or not is very strict and arbitrary from the point of view of informal nonclassical logics, so again, there are much more things outside.

Quaternaryfield and n-aryfields for every finite, infinite or post-infinite n, are defined analogously and all contain much less than the next member of the sequence. Collectively, that verse is called Platofield, and contain much less than absolutely everything.

Prismgate: it is a conceptual, informal operation which inputs concepts and outputs their direct (formal and informal) generalizations; the Prismgate operation can be repeated several times to achieve more generalized results. Prismgate can also use as input itself, resulting in generalization of the operation.

Prismfield: is a verse which contains everything that can be achieved with a Prismgate, starting from a basic object or a concept (such as a potato) and applying the Prismgate operation an amount of times which is equal to the least number Prismgate cannot properly comprehend, and, as a consequence, cannot generalize. [Prismfield is smaller than Platofield but greater than all small n-aryfields]

Schemafield: contains all information and everything accessible through manipulations of information (these manipulations are called schemas). It is greater than Platofield because some information isn't related to other information via any

means which include n-aryfield-type negations, as such negations are a special case of schemas.
[Note that Schemafield contains its own negation because negation is a schema]

More precisely, schemas are operations on information. Both schemas and information are systems which are formal or have such degrees of informality that they don't surpass the supremum of the degrees of informality present in n-aryfields. An example of informal information is the list of the least interesting books written on Earth, and an example of an informal schema is the process of sorting books by genre.

Abfield: contains everything inaccessible through information and schemas, it is much greater than Schemafield because the requirement of being accessible through information and schemas is a limitation.

Abfield can be seen as the negation of Schemafield which has an informality degree higher than degrees which Schemafield can express.

Now we are entering the high-tier Hypercosmology. This Tier is full of informal, inconceivable and transcendental concepts. At this point, difficulty of establishment of even purely informal models is a serious issue.

This tier of Hypercosmology introduces a very strong tool which can be used to define very large verses or justify the existence of seemingly nonsensical concepts. This tool is called maximal paradox resistance (MPR for short).

MPR can be viewed as a form of omnipotent logic which governs absolutely everything. MPR allows existence of all sorts of structures as it guards them from being collapsed by any paradoxes, *no matter how severe or of what nature they are*.
Levels of transcendental paradox resistance which are comparable or equal to maximal are called GPR (godly paradox resistance).
Neither MPR nor low-levels of GPR can be achieved within verses/concepts without their direct postulation, unless some form of absolute is involved within their definitions.
[Don't confuse this with mere paraconsistency, which deals *formally* with *very specific* kinds of paradoxes, so it is, out of comparison, weaker and is sometimes reachable from below]

If some property or quality of a structure is guarded by MPR, it is traditionally notated as "it is defined as X, MPR", where X is the property/quality which is guarded.

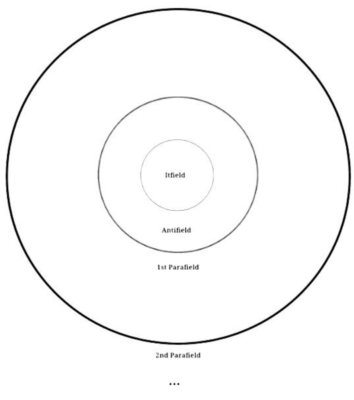
Itfield: contains absolutely everything which doesn't prevent its containment via any level of GPR. It is transcendentally larger than previously defined verses, very informally, it contains everything which is not deliberately and sophisticatedly defined to be not within it.

Parafields:

- Itfield is defined as 0th Parafield
- 1st Parafield contains 0th Parafield and everything which prevents its containment due to utilization of the least (1st) level of GPR

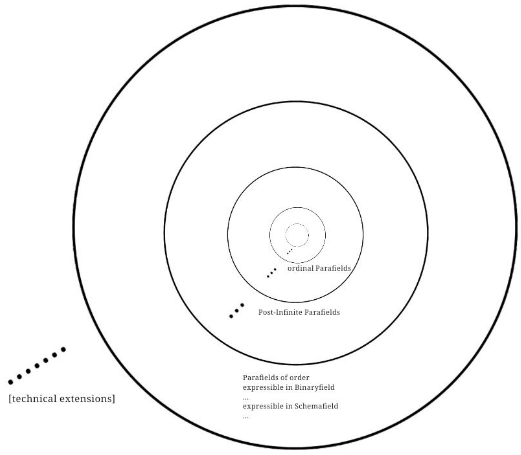
[A structure known as Antifield can be defined as Parafield with Itfield excluded from it]

- n+1st Parafield contains n-th Parafield and everything which wasn't included in n-th Parafield due to utilization of n+1st level of GPR.



[Parafields form a hierarchy similar in construction, but much greater in scale than n-aryfields. Difference between each Parafield is greater than the difference between Binaryfield and Platofield.]

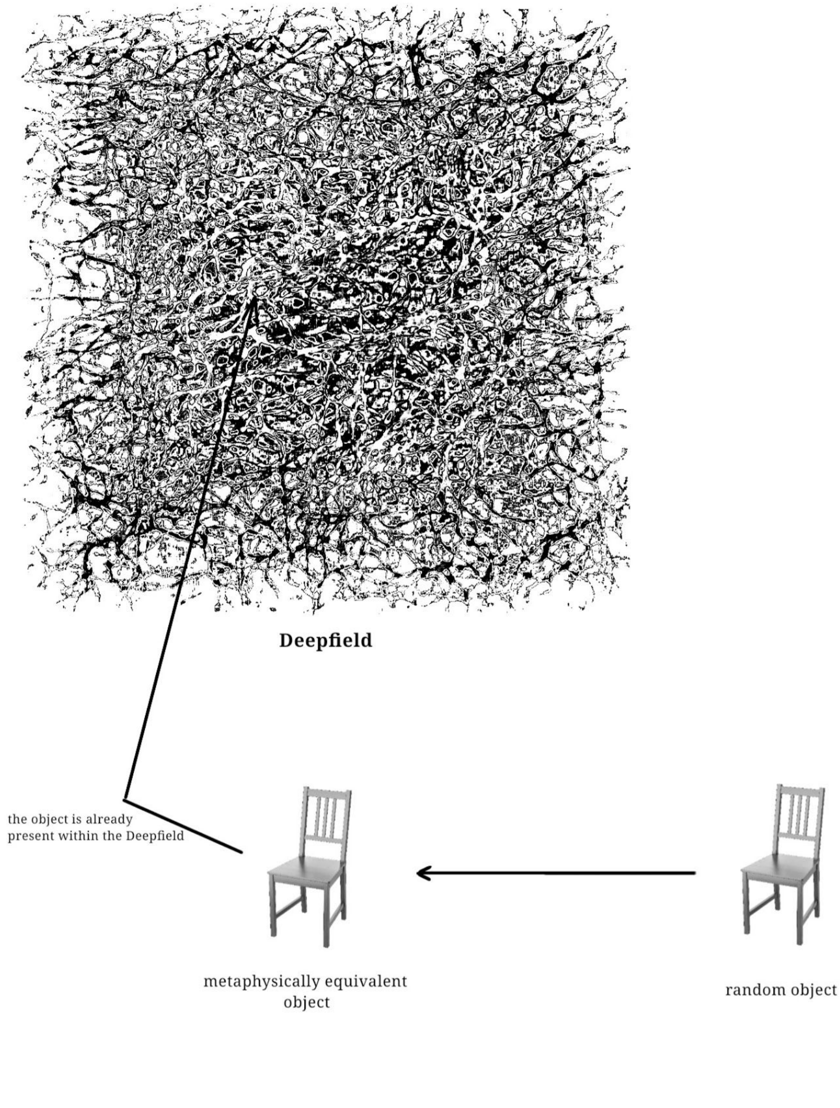
The hierarchy of Parafields can potentially be extended to MPR-utilizing Parafield, which is equivalent to Absolute Everything (as a distinct structure), but canonically, the hierarchy is limited by the least Parafield which this hierarchy approaches without necessarily reaching it.
The level of this limiting Parafield is the least number not "formalizable" in terms of the contents of this smaller hierarchy.



the Greatest Parafield, order of which is comprehendible from below, the limit of canonical Parafield hierarchy

Nonfield: contains everything which *isn't* (not *can't be*) contained, regardless of the reason why. It is greater than any (reachable) Parafield because the property of being contained is a limiting restriction.

Deepfield: the smallest thing which is too large to be increased by containing more concepts than it already does, due to already containing their structure. It is so large that it would remain unchanged even if we added a Universe, Nonfield or even the totality of Absolutely Everything into the Deepfield.
It may seem like Deepfield is equal to Absolutely Everything, however that is not true. A very basic example of what is larger than Deepfield is the following structure: "Deepfield + a concept which avoids being already contained by Deepfield, MPR"
The vast majority of concepts do have such properties, as strange as that sounds, because concepts which don't have them are limited in scope and variation by their necessary embeddability into the Deepfield.



Now we are entering the maximal-tier Hypercosmology. The level of completely transcendental absolutes such as the Omnipotent entity or Absolutely Everything

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Absolutely Everything (called the Box in the V&D community) contains absolutely everything, even the structures which prevent their containment, MPR.
[Absolutely Everything contains many different versions of itself among other very peculiar things; it also contains all of the following structures below]

S P A C E (also notated as Microbox): contains everything which is in a version of the Box which is to be considered our **local**, meaning that it directly contains *our* Universe and things which extend it. S P A C E is the least Hyperreflective verse.

Hyperreflective Verses: verses which closely resemble Absolutely Everything, meaning that an insightful expression can be made about the difference in size between the verses and Absolutely Everything, in which the description of the difference *itself* doesn't refer to any absolutes directly or indirectly. Examples include "twice as small as Absolutely Everything", or "is by several orders of magnitudes smaller than Absolutely Everything", or even "INF_ALL levels of cardinality lower than Absolutely Everything"

Almost Maximal Verses: verses that are extremely close to Absolutely Everything. These verses are usually defined as Absolutely Everything with something non-hyperreflective subtracted from it. A well-known example of such a verse is Cheesebox.

Cheesebox: Absolutely Everything without a single cheeseburger, MPR.
MPR is essential to the definition because the cheeseburger is intended to be outside of the Cheesebox under any, even highly paradoxical, circumstances.

◦ It may seem that several Hyperreflective or Almost Maximal verses collectively contain more than Absolutely Everything. However, countless amounts of Hyperreflective and Almost Maximal verses are properly contained within Absolutely Everything. This phenomenon is due to the paradox-overcoming nature of Absolutely Everything; its MPR ensures that it is always at the top of the containment hierarchy.

Numbers can also be hyperreflective or maximal:

- Godly Numbers: largest numbers via utilization of levels of GPR; (higher levels of GPR override lower levels and so, a proper hierarchy is formed).

(Within the collection of all numbers, the *vast* majority of numbers are either directly godly or derived from godly (such as 1/godly, negative godly, etc… Numbers which aren't in any sense associated with godly are an *extreme* exception)

- CAI: the largest number, via utilization of MPR

[contrary to popular misconception CAI has nothing to do with Cantor's Absolute Infinity, which is arguably equal to Ord]

It can be said that Absolutely Everything contains CAI objects while Hyperreflective verses contain a godly amount, though such comparisons at this level are not entirely straightforward. As a consequence of that, hyperreflective and godly are essentially synonymous, it is simply a tradition to associate the word "hyperreflective" with verses and "godly" with numbers and levels of extremely high paradox resistance.

Now we are going to explore the opposite direction, the hierarchy of small structures which are smaller than fundamental particles of our Universe. The small-scale hierarchy will be explored much more briefly due to most levels being trivially derivable from the associated levels of the large-scale hierarchy. Some levels, however, are skipped due to being much less intuitive and fruitful than their large-scale counterparts. The following section is ordered by reduction in size.

The hierarchy of small concepts is a direct reflection of the hierarchy of large concepts. A very simple way of deriving small particles is through the V&D concept of voids. A void of a concept or a verse is the fundamental particle out of which all of its parts can be constructed. The bigger the verse is, the smaller is its associated void counterpart.
[Due to the unfortunate naming conventions, V&D voids are often misinterpreted as empty spaces or abysses; in fact, those have nothing to do with the V&D concept of voids]

Voids of concepts are notated as "Xvoid" where X stands for the original concept.

It is difficult to say what local Universevoid is, due to incomplete human understanding of physics. It may be quantum foam or strings from string theory, or something much, much smaller.

Multiversevoid is a hypothetical fundamental structure which composes all physics within a Megaverse. Archversevoids (often shortened to Archvoids) are defined similarly.

Selfversevoid (often shortened to Selfvoid) is the fundamental particle of a Selfverse. Selfvoids are as much smaller than themselves as their respective Selfverses are bigger than themselves.

(Logic-Verse)voids: the fundamental particles of the everything which satisfy certain systems of formal logic.

Maiorversevoid (often shortened to Maiorvoid) is the fundamental particle of the Maiorverse, it is so much smaller than itself that its degree of self-subsidence is the supremum of all post-infinite numbers.

Formalvoid: the fundamental particle of everything formal by nature.

Schemavoid: informational nothingness.
...
[Past this point there exists a very large gap within the hierarchy, voids of large verses do not have non-technical characterizations, intermediate small concepts may be more intuitive and fruitful, but none are yet discovered]
...
S P A C E-void: the largest inverse-hyperreflectional (demonic, as an opposite to godly) verse.
...

We are now entering the realm of near-minimality, this realm is full of peculiar paradoxes. The following section is ordered by increment in size.

Nothing: has no properties, internal or external. It is far below even the concept of containment itself.

Sergepoint: the fundamental unit of reality, as a consequence it is the fundamental unit of every variation of Absolutely Everything. It is the smallest real object. All objects, properties and concepts in general can be expressed as a whole (finite or not) amount of sergepoints, even such peculiarities as "the property of being irreducible to sergepoints".

Basic Non-Concepts: small, finite amounts of sergepoints which are too small to have any sensible properties apart from their sergepoint amount.

Conceptual Nothing: the smallest amount of sergepoints which isn't enough to characterize any sensible properties

Conceptvoid: the smallest amount of sergepoints which has the property of having sensible properties.

Basic Concepts: amounts of sergepoints large enough that sensible properties emerge in them.

box postfix notation: "X"box is Absolutely Everything with X removed from it, MPR. X stands for any concept.

Boxbox: Absolutely Everything with itself removed from it, MPR
[A common misconception is that Boxbox is Nothing, however, that is false. Boxbox has many external properties, so it is greater than most Basic Concepts.]

Boxboxboxbox is equivalent to ((Boxbox)box)box), which is Absolutely Everything with "Absolutely Everything with Boxbox removed from it" removed from it. It is slightly greater than Boxbox, and generally, concepts notated as Box with larger odd numbers of concatenated "box" are successively slightly larger.

The next section is about number systems which are useful in hypercosmology.

Previously we defined transfinite ordinals as the position of elements in sets. Both positions and elements of sets are seen as strictly whole (integer-like) numbers. As a consequence, ordinals themselves are incapable of capturing conceptions such as $1/\omega$ or $\omega+0.5$

Even whole numbers such as $\omega - 1$ make no sense as ordinals because there is no position which comes right before ω. Numbers like ω, $\omega+\omega$ or ε_0 are called limit ordinals as they are a limit of ordinals, as they aren't successors to any concrete ordinal, but are the supremum of an infinite amount prior to them.

Fortunately, an extension of the concept of ordinal numbers called surreal numbers formalizes such expressions.

Surreal Numbers has opposite and inverse elements to any number except 0; it contains all the reals and it is in agreement with their arithmetic and their formal theory in general, but as opposed to the field of Real Numbers, the field of Surreal Numbers is enriched by infinite numbers; it contains all the ordinals, their inverses, and generally, any operation which is defined on all reals in the theory of real numbers, when applied to ordinals, results in a valid surreal number. As a consequence, expressions such as $\omega - 1$, $\omega / 2$ or even $\sqrt{(\omega+53)}$ and log_2^(ε _0) gain meaning.

The following hierarchy of numbers is the hierarchy of positive amounts; just like the hierarchy of verses, it has 2 parts which reflect each other - the whole numbers and their inverses. The neutral position in the hierarchy is the number 1. This hierarchy only mentions whole positive numbers and their inverses, however it can be extended in a manner similar to the manner in which the field of Surreal Numbers extends the field of Real Numbers.

The hierarchy can be extended even further invoking number systems such as Complex Numbers or Hypercomplex Numbers, at the cost of the hierarchy losing any linearity even in the transcendental sense.

In order of ascension:

Absolute Zero (also called the Minimal Number) is the minimal number, MPR. It is associated with its opposite which is the maximal number, MPR

Demonic (Inverse-Godly) Numbers: Numbers which are transcendentally small, the largest such number is 1/(N U M B E R) [see below]

Inverse Xeno-Infinite Numbers: the numbers so small that they cannot be formalized even in theory. The largest such number is 1/Formalnumber, this number is the largest positive number which cannot be distinguished from Absolute Zero by any *formal* system.

Inverse Post-Infinites: the numbers so small that they violate ordering principles, the largest of these is 1/E_0 which is closer to 0 than 0 itself.
[Formal systems as we know them are incapable of formalizing this number]

Proper Collection Infinitesimals: numbers of the form: 1/(Proper Collection-Ordinal)

Infinitesimals: numbers of the form 1/(Transfinite Ordinal)

Rational Numbers of the form 1/(Natural Number)

1

Natural Numbers

Transfinite Ordinals, starting from ω

Proper Collection-Ordinals, starting from Ord

Post-Infinite Numbers, starting from E_0

Xeno-Infinite Numbers: numbers which are too big to be formalizable, even in theory. The least Xeno-Infinite number is Formalnumber which is the number of Formalvoids in Formalverse.

Godly Numbers: numbers which are transcendentally large; these numbers start from N U M B E R (the number of S P A C E-voids in S P A C E)

CAI: the maximal number, MPR. This number is associated with Absolutely Everything, Absolute Omnipotence as well as other absolutes.

The next section is about entities which reside within hypercosmological verses.

Entities are generally temporal. They function via means of causal relationships. For example, willing is a process which begins at a demand and ends at fulfilment. If no causal relationship can be established within a verse, then no entities can inhabit it. Causal relationship can relatively easily be established within verses with a single temporal dimension. It is already much more difficult to establish such relationships in verses with two or a small finite amount of temporal dimensions. After that the conceptual difference between causal and other types of relationships starts to blur and it is difficult to distinguish entities from inanimate objects due to things becoming increasingly abstract.

Regular hypercosmological verses are known for being extremely abstract and as a consequence of that, definitions of sensible entities of their level are extremely problematic.

One could artificially define verses of hypercosmological scale with one or two temporal dimensions but such verses are a minority and barely capture the essence of hypercosmological mechanics and relationships. As a consequence of that entities within such verses are of no essential difference from universal or multiversal beings, except for the difference in scale. Due to this reason proper hypercosmological entities are hardly ever defined. There are however, a few exceptions.

The True God (often shortened to TTG) is V&D's Absolutely Omnipotent entity, which is synonymous to an entity that is maximally powerful. It is highly abstract, essentially, its only definite property is being Absolutely Omnipotent.

All Absolute Omnipotents are equivalent because any differences could potentially lead to a subtle lowering of the power-level of such entities. For example a benevolent or malevolent Absolute Omnipotent could be limited by his own ethical system; an Absolute Omnipotent with several eyes could be limited by not having a different amount of eyes by default. Such limitations might seem virtual but they could potentially cause a loss in a fight between the Absolute Omnipotent and Nigh-Omnipotents.

Nigh-Omnipotents: these entities are the entities which are comparable in power to the Absolute Omnipotent. They relate to the Absolute Omnipotent in the same way godly numbers and hyperreflective verses relate to their maximal counterparts - CAI and Absolutely Everything.
Just like 3 (or more) hyperreflective verses half the size of Absolutely Everything are in fact collectively smaller than it, 3 (or more) Nigh-Omnipotents with half the power of the Absolute Omnipotent are, in fact, collectively weaker.

The weakest entity which is Nigh-Omnipotent is called D E A T H, which relates to the Absolute Omnipotent in the same way S P A C E relates to Absolutely Everything.

Within Absolutely Everything, highly paradoxical regions exist where there can be 2 versions of the Absolute Omnipotent in a fight. The result of such fights by default is both winning, which means that they turn the reality around them in such a way that both are simultaneously perfectly satisfied.

The next section is about Googology, which is an occupation similar in spirit to Recreational Cosmology

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Googology is the study and nomenclature of large numbers. One who studies, and invents, large numbers and names of large numbers is known as a googologist. A mathematical object relevant to googology is known as a googologism

Googologisms are numbers which are too large to be represented in any physical way within our observable universe. Googol, the number which is equal to 10^100 is traditionally considered to be a googologism, even though the volume of the universe in Planck lengths (the smallest physical unit of measurement of distance) is approximately equal to 4,65 * 10^185.

Googologisms include numbers such as 10^10^10^100^200^300 or 100^100^100...(repeated a hundred times)

Unlike previous examples, most googologisms are too large to be physically represented via nested exponentiation function even if our observable universe was full of paper and ink. To deal with this issue functions which grow much faster than exponentiation are defined.

The most natural hierarchy of such functions is the hierarchy of hyperoperators. The hierarchy is defined recursively:

- The 1st hyperoperation is addition
- The 2nd hyperoperation is multiplication (a * b = a + a + ...(b times))
- The 3rd hyperoperation is exponentiation, (a ^ b = a * a * ... (b times))

and generally,

- n+1st hyperoperation is defined as repeated application of nth hyperoperation (a [hyp. n+1] b = a [hyp. n] a [hyp. n] ... (b times))

The 4th hyperoperation is called tetration, and is the most well-known hyperoperation. Tetration is notated with two up-arrows ↑↑. The next hyperoperation is called pentation and is notated with three up-arrows ↑↑↑. Further hyperoperations are notated likewise.

There are functions which grow faster than any hyperoperation. Definitions of such functions usually involve diagonalization of the hierarchy of hyperoperations. A function we will call Q(n) is provided as an example of such a function, defined as: Q(n) = (n [hyn. n] n)

So, for example, f(3) = 3^3 and f(4) = 4↑↑4 etc...
It is not hard to see that it outgrows any fixed hyperoperation at significantly high input values. This phenomenon is called *eventual domination*, it is a way to measure how fast a function grows.

Faster functions are defined using hierarchies which utilize countable ordinals, those hierarchies' definitions being more technical, so we will not go into the details. Roughly speaking, most such hierarchies associate a countable ordinal with a function. The greater a countable ordinal is, the faster the function is.

The Church-Kleene ordinal notated as ω1ck, which is the least non-recursive ordinal, plays a special role in such hierarchies. It is the supremum of all recursive ordinals which are associated with *computable functions.*

A function is computable if a computer program which calculates it is, at least in theory, constructible. Some functions are incomputable because they grow too fast to be computable with present day hardware. Such functions grow incomparably faster than computable functions. Hardly anything can be known about the values of such functions as it is presently impossible to construct a program which calculates the functions' values for each and every natural number. However, we still can compare them using the notion of eventual domination.

The fastest functions ever defined (all of which are incomputable because of their growth speed) in order of increase in speed are:

- the Busy Beaver function notated as BB(n)
- the Rayo function notated as Rayo(n)
- the BIG FOOT function
- the Little Biggedon and the Big Biggedon (Sasquatch)
  [which are currently not understood well by the googlogical community]

The above functions are mathematically strict and well-defined, however googology can also be extended into the informal realm. This is frowned upon by the general googological community as the functions lose the quality of being mathematically meaningful and precise, but in a hypercosmologically-flavored way, these functions can be considered to be even faster than regular uncomputable functions. Examples of such functions are the following functions:

Oblivion(n): the largest number defined using no more than n symbols in a K(n) system", where a "K(n) system" is a "complete and well-defined system of mathematics that can be described with no more than n symbols".

Utter Oblivion(n): the largest finite number that can be uniquely defined using no more than a Oblivion(n) symbols in some K(Oblivion(n)) system in some K2(Oblivion(n)) 2-system in some K3(Oblivion(n)) 3-system in some K4(Oblivion(n)) 4-system in some ....KOblivion(n)(Oblivion(n)) Oblivion(n)-system where the number Oblivion(n) can be represented with one symbol/byte", where a Km(n) m-system is an arbitrary well-defined system of mathematics that can generate K(m-1)(n) (m-1)-systems, and which can be uniquely described in, at most, n symbols, and a K1(n) system is an arbitrary, well-defined system of mathematics, which can be uniquely described in n symbols.

Greater informal functions can, of course, be defined.

Chapter 2
Metaboxiality

It is widely acknowledged that nothing is above Omnipotence. Any attempt at defining an entity which is mightier than Omnipotent is caused by the misunderstanding of the term. Obviously, even if the defined entity succeeds at being mightier than the Omnipotent, it only means that the Omnipotent wasn't actually Omnipotent. Omnipotence is, by definition, always at the top.

The property of always being at the top no matter what also belongs to Absolutely Everything. If something was outside of Absolutely Everything then the so-called Absolutely Everything wasn't in fact Absolutely Everything. The same principle applies to CAI - no number can be above it, and if some number succeeded at surpassing it, it wasn't actually CAI which was surpassed in the first place.

Objects with this property are called absolute or, synonymously, maximal. No maximal object can be surpassed. One may attempt to define objects which are "twice as great as maximal" or "infinitely greater than maximal," but those are bound to be failed attempts.

However, the previous paragraph is, actually, an act of *speculation*. One cannot logically prove that things beyond Omnipotence, CAI or Absolutely Everything do not exist as those concepts are above logic.

One can try to argue from the point of definitions. Since definitions of Omnipotence, CAI and Absolutely Everything directly state that nothing could be above them, one can deduce that if something was in fact above the so-called absolutes, then the definitions were misapplied, and the things above are now the candidates for being the actual absolutes. However, even that is a form of speculation, because one cannot prove using the concept of definitions that things cannot be above the *concept* of definitions.

One can argue that constructs such as things which are above the concept of definitions are ill-defined or completely nonsensical, but one cannot ultimately prove it. One can only believe that to be the case.

Absolutists (in V&D use) are people who believe that absolutes cannot be surpassed and that things which surpass absolutes are always either failed attempts or meaningless.

Transcendentalists (in V&D use) are people who believe that there are things which are valid and surpass absolutes.

Agnostics (in V&D use) are people who are open to both possibilities and restrain from taking a side in the argument.

Some people believe that some absolutes are surpassable, and some absolutes are not. For example, one may believe that Absolutely Everything is surpassable and Omnipotence isn't. Because of this and the fact that V&D is centered around the theme of verses, more specific terms are used:

- Box Absolutism: the belief that nothing is outside of Absolutely Everything
- Box Transcendentalism: the belief that some concepts succeed at being truly outside Absolutely Everything, and are beyond the scope of MPR.

Concepts which are truly outside Absolutely Everything are referred to as metaboxial. Box Absolutists believe that the term has no meaning as it is referring to nonsense.

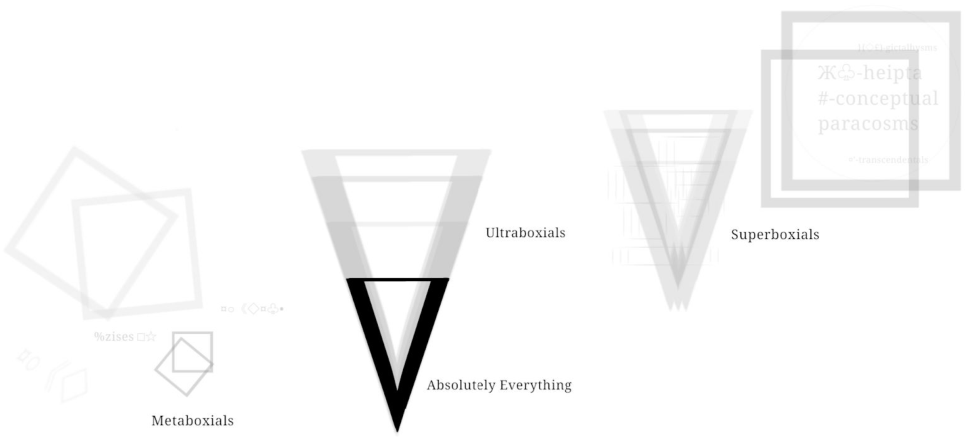
There are structures which Box Transcendentalists consider to be metaboxial. Box Absolutists tend to view those structures as falsely metaboxial (as in actually, within Absolutely Everything) or ill-defined.

There are three main types of metaboxial objects:

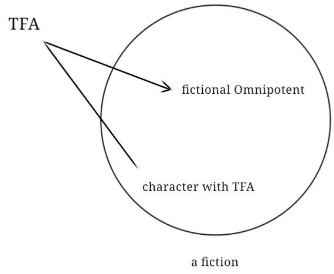
- Metaboxial - the most general category, simply stands for ‘truly outside Absolutely Everything’

- Superboxial - Metaboxial, which is also truly larger than Absolutely Everything in scope, but not necessarily containing Absolutely Everything itself

- Ultraboxial - Superboxial while also containing Absolutely Everything

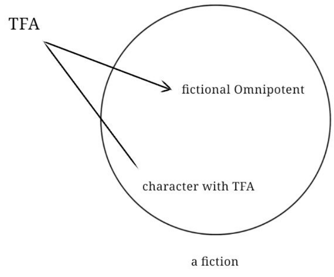


A work of fiction can be viewed as a fictional version of Absolutely Everything. Author of the story is in full control of everything within it, even the fiction's Omnipotent entity is at the mercy of the author's will. This phenomenon is known as Author Authority.



The fact that author of the story is greater than fictional omnipotent proves that the act of writing fiction isn't an act of referring to a part of Absolutely Everything but is the act of metaphysical postulation of some realm which is below the real-life version of Absolutely Everything.

One of the most well-known means of transcending absolutes is via the notion of Transfiction. Transfiction is a phenomenon of fictional characters being able to transcend their own work of fiction by leaving it and entering reality. That is a feat which even Omnipotents characters can't achieve. A stronger version of transfictionality s called Transfictional Author Authority (shortened to TFA). Characters with TFA are able to edit their own fiction's plot. they are by default greater than other fictional characters, even Omnipotent ones, as they can force Omnipotents to lose to them in a fight by writing that event as a part of their story.

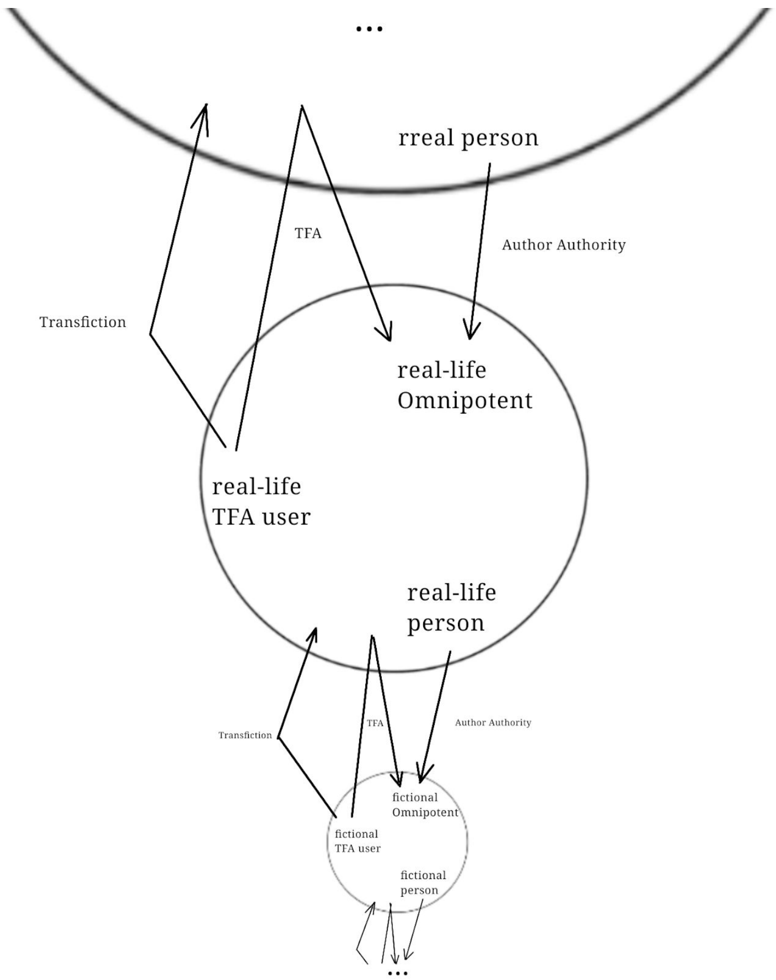


Transfiction isn't necessarily just a fiction-related term. A transfictional entity from the real world may transcend reality and enter a realm so great that it views our reality as a work of fiction. It is very important to note that reality in this context refers to the *real-life* Absolutely Everything, and the greater realm refers to a superboxial structure called RReality.

RReality is a realm which is so great that it treats what we call "real-life" as just a work of fiction. RReality contains greater than CAI amount of alternative real-life realities. It also contains the *rreal* version of Absolutely Everything. Whether RReality is ultraboxial or only superboxial depends on whether our real-life is authored by some entity within the rreal version of Absolutely Everything. Either ways, rreal entities are beyond-Omnipotent because they have Author Authority over the real-life Omnipotent.

What we call real-life is a fraction of a *rreal* sergepoint.
[Similarly, what fictional characters call real-life is a fraction of a real sergepoint]

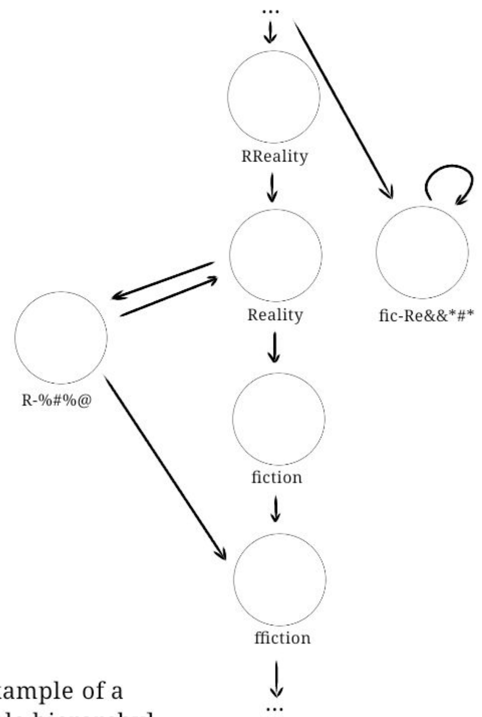
RRReality, RRRReality, … ; ffiction, fffiction… are defined analogously.



Horizontal or higher-order TFA can also be defined:

- Horizontal TFA is TFA between different realms of the same level
- Higher order TFA is a variant of TFA which is capable of transcending several levels at once, for example going from fiction directly to RReality. it is notated as nTFA where "n" is the difference between the levels.

R-levels are not necessarily ordered in a strict hierarchy. Some R-levels may be parallel to each other, circular or structured even more peculiarly.



[an example of a possible hierarchy]

RR……(CAI R's)…eality is greatly superboxial but still can be transcended via means of TFA. An example of such a verse is
RR….(amount of real-life worlds in RReality)…eality
The limit of *all* R-levels is called Authorlock.

Nothing (in V&D use) can also be viewed as a form of an absolute. It is absolutely the least thing within Absolutely Everything. However, TFA is unable to transcend this absolute. Greater R-level R…ealties have as their void variants smaller fractions of sergepoints, but even Authorlockvoid is greater than Nothing. This proves that sub-Nothing concepts require metaboxial principles different from TFA.